# **Research Technical Paper**

# The Real Interest Rate Spread as a Monetary Policy Indicator

Frank Browne Mary Everett \*

Central Bank and Financial Services Authority of Ireland P.O. Box 559, Dame Street Dublin 2 Ireland http://www.centralbank.ie

<sup>\*</sup>The authors are, respectively, Head of the Bank's Monetary Policy and Financial Stability Department and an Economist in the Bank's Statistics Department. The views expressed in this paper are the personal responsibility of the authors. They are not necessarily held either by the CBFSAI or the ESCB. The authors would like to thank John Frain for invaluable assistance. All errors and omissions are the sole responsibility of the authors. Email: frank.browne@centralbank.ie, mary.everett@centralbank.ie

# Abstract

This paper employs a consumption-based capital asset pricing model to derive the generalised Fisher equation, in order to estimate the natural rate of interest and corresponding real interest rate spread for the US. Analysis reveals not only is the estimated real interest rate spread a useful measure of the degree of looseness/tightness in the Federal Reserve's monetary policy stance, but also the variable contributes substantially to an understanding of the evolution of US inflation over the period 1960-2005.

#### 1. Introduction

The idea that interest rates are the only channel through which monetary policy actions are transmitted to the real economy and inflation is one which is increasingly in the ascendant among academics and practical central bankers alike. This is reflected in the de facto downgrading of monetary aggregates as indicators by almost all central banks.

However, the nominal interest rate in itself may not provide an anchor for monetary policy. Let's suppose that the central bank is targeting a particular level of the short-run nominal interest rate. Now imagine an adverse supply shock or an inflation scare occurring. This prompts people to expect inflation later. This expectation of inflation drives down the real rate of interest and stimulates aggregate demand and actual inflation. This further reduces the real interest rate generating additional demand and inflation, which, in turn, further depresses the real rate given a nominal rate pegged down by monetary policy. Here we have a case of expected inflation giving rise to actual inflation, which, in turn, generates further inflation later. A central bank's commitment to a nominal interest rate target could therefore generate an acceleration in inflation since the money supply is allowed to expand passively to facilitate the emerging inflation. With the demise of monetary targeting and even the downgrading of monetary reference values, there is a need for some interest rate reference value such that deviations of interest rates from this value signal the emergence of inflation later and hence provides a reliable guide to the current stance of monetary policy.

The inflation indeterminacy just noted is essentially a description of Wicksell's famous "cumulative process" in which the misalignment of the actual real interest rate (controlled by the central bank via its leverage over nominal rates of interest) from its corresponding equilibrium value drives nominal demand above the ability of the economy to supply at prevailing prices. The resulting inflation only ceases when the maladjustment of the actual real rate is corrected by monetary policy action.

In the context of this increasingly dominant interest rate paradigm of monetary policy, the spread between the actual real rate of interest, which the central bank can control and the corresponding "natural" rate, also known as the "equilibrium" or "Wicksellian" real rate, which is determined by the optimising behaviour of private agents and which is exogenous to the monetary policy actions of the central bank, could provide a valuable indicator (or reference value) for the central bank in formulating its monetary policy. Woodford (2003) is probably the most prominent advocate of this spread as a key variable in examining inflationary or disinflationary (deflationary) pressures in the economy. In a recent interview in "Newsletter", Study Center Gerzensee, January 2005, he says that: "…understanding variations in the natural rate of interest ought to be of great practical importance. There is almost no research on trying to implement that empirically and to track variations of the natural rate of interest in real time".

In a world in which it is generally believed monetary policy is increasingly transmitted to the real economy and to inflation via interest rates rather than via nominal quantity variables such as the money stock or credit aggregates, Woodford is clearly correct in arguing that it is a matter of growing importance for central banks to have such a real interest rate spread indicator in its analytical toolkit to be used in formulating monetary policy.

Officials of the Federal Reserve are making increasing reference to a neutral interest rate for the economy and have recently indicated a need for the federal funds rate of interest to rise to this level, which apparently is deemed enough to curb inflationary pressures without compromising growth. However, there is a lot of uncertainty about what this neutral level of the interest rate is. In July of 2004, the Federal Reserve chairman, Greenspan, conceded that he did not know what the level of the neutral rate was: "Actually, we don't know what neutrality is until we get there".

An extract from a recent meeting of the Bank of England's Monetary Policy Committee indicates that it "...discussed whether it was helpful to think about the appropriate level of nominal interest rates by reference to the concept of a 'neutral' level which provides neither stimulus nor restraint to the economy". The conclusion of the discussion was that "....some members of the Committee found the concept of the neutral rate useful in deciding the interest rate policy, other members found the uncertainty surrounding its level so large that the concept was of little use as a practical guide to policy". Both quotes were taken from Neiss and Nelson (2001).

The uncertainty about the level of the interest rate that corresponds to the neutral rate is no doubt related to the fact that the derivation of such a neutral rate is far from straightforward. It involves tricky estimation. The derivation of the *actual* real rate of interest is fairly straightforward. It merely involves subtracting expected inflation from the actual observed nominal rate, where expectations of inflation, although not directly observed, are estimable. The natural rate, on the other hand, presents much greater difficulties. It is not directly observed and its estimation is model dependent in the sense that there is no clear-cut consensus on what are the driving forces behind movements in the natural rate.

The objective of this paper is to report our work in estimating the natural rate, and the corresponding real interest rate spread, employing one particular model and using data for the US economy. If monetary policy makers knew, if only approximately, what the number was for the natural real rate of interest, then in order to achieve a neutral stance for monetary policy, they could set the short-term nominal rate of interest (i.e., the de facto monetary policy instrument or a rate very closely linked to this) such that the corresponding actual real

rate equalled this natural (or equilibrium) real rate. Under such circumstances, but subject to a few further restrictive assumptions, this rate would yield price stability without at the same time restraining growth.

The economy would be subject to neither inflationary nor deflationary pressures coming from monetary policy itself if this rate prevailed and, accordingly, no long-term inflationary or deflationary pressures since, over this horizon, these all come from monetary policy. However, when the economy is subject to shocks that cause incipient inflationary or deflationary pressures in the short to medium term, the central bank may have to alter the spread temporarily so as to bring the economy back to equilibrium.

Other measures of aggregate demand-supply imbalances commonly used, such as the output gap and the deviation of unemployment from its NAIRU level are, arguably, too far removed from the monetary policy instrument typically deployed by the central bank, namely the short-term rate of interest, to be timely, or even reliable measures, of the monetary policy stance. Furthermore, interest rates unlike these other measures are, arguably, less subject to measurement error and are almost never revised. It has also been argued that the response of the natural rate of interest to shocks is flatter than the response of potential output to shocks, all of which suggests that the real interest rate spread may be a more reliable indicator of inflationary pressures and a more reliable anchor of the stance of monetary policy.

The plan of the paper is the following. The next section (i.e., section 2) looks briefly at what theory has to say about the equilibrium (or natural) rate of interest and introduces the consumption-based capital asset pricing model (CCAPM), which is the theoretical foundation for the derivation of the natural rate, which we are using. This model is then used (in section 3 of the paper) to derive a generalisation of the famous Fisher equation relating the nominal interest rate to the real interest rate and inflation by placing this relationship in a stochastic

setting. In section 4 we proceed to use this approach to show how the various components of the nominal interest rate according to the CCAPM model (i.e., the discount rate, expected consumption and inflation, the respective variances of these two variables and the covariance between these expected consumption and inflation) can be isolated and estimated separately. It also demonstrates how they can be assembled to obtain an estimate of the equilibrium real rate of interest, the variable of greatest interest to us in this paper. Section 5 goes on to consider the role of money and monetary uncertainty impinges on this relationship. In section 6 a few additional considerations are examined such as the maturity of the interest rate used in the analysis and changes in the monetary-policy operating framework of the Federal Reserve especially in the late 1970s and early 1980s which induced huge volatility in the short-term interest rate. The results from estimating the model are reported in section 7. At this stage we also look at alternative specifications that have been considered in the literature (which involve including exchange rates and a number of macroeconomic variables as additional determinants of the nominal interest rate). Our estimate of the equilibrium rate is then discussed. We conduct a visual inspection of the difference between the actual real rate of interest and our estimate of the natural rate (real interest rate gap) to see what kind of job it does in explaining inflation developments in the US since the early 1960s. We conclude that its performance is quite impressive and suggest that this type of real interest rate gap could be a useful indicator for monetary policy purposes. The overall conclusions of the paper are in section 8.

#### 2. What does Theory Have to say on the Equilibrium Rate?

In estimating the natural rate of interest, one must first be guided by what theory has to say with respect to the natural rate. Wicksell's starting point was the quantity theory of exchange, which says that, in a world of paper money controlled by the central bank, the price level will tend to be proportional to the money stock. His cumulative process was designed to explain the mechanism by which this relationship between money and prices is established in practice. An expansionary monetary policy, for example, depresses the real market rate of interest relative to the natural rate boosting aggregate demand beyond supply and generating inflation. The market rate adjusts back to the natural rate as the increase in the price level lowers the real money stock back to its original level. If the banking system, in conjunction with the central bank did not create money, then the market rate would always be equal to the natural rate.

We need some operational definition of the natural rate in order to estimate it. It is the rate that would be observed if the economy were "classical" in the sense of being free of nominal frictions and informational asymmetries. In such a world, households would be able to allocate consumption across time as desired with the only constraint being the cost of doing so which is the natural rate itself. The model on which we base our estimate of the equilibrium rate is founded on this observation. Household behaviour in such a world can be captured by the so-called capital asset pricing model (i.e., CAPM). The specific version of this model, which we employ here, is the consumption-based CAPM (i.e., the CCAPM). Using this approach it is possible to infer from the actual observed short-term nominal interest rate what the natural real rate of interest would be in the absence of the types of frictions actually observed.

In other words, the natural or equilibrium real rate can be inferred from the portfolio behaviour of the private sector using this asset-pricing model. The consumption-based CAPM is used to capture intertemporal portfolio behaviour of households and, in the process, can identify the various components of the actual nominal rate of interest that have to be isolated in order to home in on an estimate of the equilibrium real rate of interest. Koedijk et al (1998) distinguish between macroeconomic explanations and asset pricing theories of the real interest rate. The advantage of the latter is a clear theoretical micro foundation and a consistent treatment of risk. The macro explanations tend to be more ad hoc, in attempting to explain interest rate movements in terms of economic state variables, and don't address issues relating to risk explicitly. Following the arguments of Cochrane (1994), Koedijk et al endeavour to blend the two approaches<sup>1</sup>. The spirit of the approach followed here is similar. In fact, we examine three types of models, i.e., the pure consumption-based CAPM with a liquidity effect as described above, this model supplemented by exchange rate considerations (i.e., the CCAPM with exchange rates), and finally a model augmented by a number of macroeconomic variables not suggested by the CAPM model (i.e., the augmented model).

The equilibrium real rate of interest is the rate that prevails in a classical world of fully flexible prices. If prices move freely following shocks, macroeconomic quantities (such as income, saving and investment) need to move less. We would therefore expect the variance of the estimated equilibrium real rate (in a world of smoothly adjusting prices) to be significantly, and probably substantially, less than that of the actual real rate (in a world of inflexible sluggish prices).

## **3.** Generalisation of the Fisher Equation

The consumption-based Capital Asset Pricing Model is used to derive the generalised Fisher equation. There are three basic steps involved in the derivation: first, an equilibrium condition between the nominal and real rates of interest is derived: an expression for the real

 $<sup>^{1}</sup>$  It has also been noted by Blanchard and Summers (1984) that ascribing high real interest rates to only one cause will not fit the facts.

rate is then established (obtained from the condition that the one period real rate must equal the ex ante marginal rate of substitution between consumption now and in the next period): this expression is then substituted into the former equation to derive the relationship between the nominal interest rate and its proximate determinants i.e., the generalised Fisher equation.

#### 3.1 Derivation of Equilibrium Condition Between Nominal and Real Rates of Interest

An individual agent maximises utility subject to a budget constraint:

$$E_t\left[\sum_{i=0}^{\infty} \Phi^i U(C_i)\right]$$

The instantaneous utility function takes the form of constant-relative-risk-aversion:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

Assume a single consumption good and that utility is isoelastic and time separable. An individual representative consumer maximises expected utility over an infinite horizon:

$$E_{t} \sum_{i=0}^{\infty} \Phi^{i} \frac{1}{1-\gamma} C_{t+1}^{1-\gamma} \qquad 1 > \Phi > 0, \gamma > 0 \qquad (i)$$

 $E_t$  – Expectations conditional on information available in period t

 $\Phi$  – Discount factor

 $\gamma$  – Coefficient of relative risk aversion

Equilibrium asset returns are established from the first order condition of the representative consumer's maximisation problem.

The first order condition is:

$$C_{t}^{-\gamma} = \Phi E_{t} [C_{t+1}^{-\gamma} Q_{t+1} / Q_{t+1}]$$
(ii)

where  $Q_t$  is the value of an asset at stated in terms of consumption goods in period t.

If it is assumed that the asset is a nominal bond with a nominal interest rate of  $I_t$  then the ex post real return on investing in nominal bonds between periods t and t+1 is:

$$(1+I_t)P_t / P_{t+1} = Q_{t+1} / Q_t$$

where  $P_t$  is the nominal price of a good at time t and where:

$$1+R_t=Q_{t+1}/Q_t$$

Therefore:  $(1 + I_t)P_t / P_{t+1} = 1 + R_t$ 

Optimal portfolio choice necessitates that expected yields on nominal and real bonds of identical maturity must be equivalent when considered in terms of expected utility. Adding expectations and the marginal utility of consumption in t+1 establishes the equilibrium condition for an individual consumer:

$$E_{t}[U'(C_{t+1})(1+I_{t})(P_{t}/P_{t+1})] = E_{t}[U'(C_{t+1})(1+R_{t})]$$

where  $R_t$  is the real interest rate on a one period bond and  $I_t$  is the nominal interest on a one period bond.  $P_t / P_{t+1}$  is the change in purchasing power of money over one period and  $U'(C_t)$  the marginal utility of consumption in period t.

The first order condition for nominal bonds is:

$$C_{t}^{-\gamma} = \Phi E_{t} \left[ C_{t+1}^{-\gamma} (1+I_{t}) P_{t} / P_{t+1} \right]$$
(iii)

Applying log normality allows equation (iii) to be rewritten as the equilibrium asset pricing condition:

$$i_{t} = r_{t} + E_{t} \Delta p_{t+1} - \frac{1}{2} Var_{t} (\Delta p_{t+1}) - \gamma Cov_{t} (\Delta c_{t+1}, \Delta p_{t+1})$$
(iv)

The expected value of  $\Delta p_{t+1}$  is not the inverse of the expected value of the change in inflation in situations of uncertainty and hence the inclusion of the term  $(1/2)Var_t(\Delta p_{t+1})$  in equation (iv).

# 3.2 Derivation of Equilibrium Real Rate

We next derive a separate expression for the risk free real rate of interest, i.e.,  $r_t$  in expression (iv). This is obtained from the condition that the one period real rate must equal the ex ante marginal rate of substitution between consumption now and consumption in the next period. This can be derived by considering the return on a real bond. If the known real rate of interest at time t is  $R_t$ , then the purchase of a real bond in period t for  $Q_t$  consumption goods entitles the holder to  $Q_t(1+R_t) = Q_{t+1}$  goods in t+1.

The first order condition for a real bond is found by substituting in (ii):

$$C_t^{-\gamma} = \Phi E_t \Big[ C_{t+1}^{-\gamma} \big( 1 + R_t \big) \Big] \tag{V}$$

Equation (v) is the equilibrium relationship between the real rate of interest and the ex ante intertemporal marginal rate of substitution. Applying the assumption of log normality and rearranging defines the log of the real rate:

$$r_{t} = \gamma E_{t} \Delta c_{t+1} - \frac{1}{2} \gamma^{2} Var_{t} (\Delta c_{t+1}) - \phi$$
 (vi)

where  $\phi = \log \Phi$ . If the future is heavily discounted (i.e., a high value of  $\Phi$ ), current consumption is greater and savings are lower.

#### 3.3 Derivation of the Generalised Fisher Equation

We are now in a position to derive a relationship between the nominal interest rate and its proximate determinants. By substituting equation (vi) into (iv) we can derive the following generalised Fisher equation:

$$i_{t} = \gamma E_{t} \Delta c_{t+1} - \frac{1}{2} \gamma^{2} Var_{t} (\Delta c_{t+1}) + \delta + E_{t} \Delta p_{t+1} - \frac{1}{2} Var_{t} (\Delta p_{t+1}) - \gamma Cov_{t} (\Delta c_{t+1}, \Delta p_{t+1}) \quad (\text{vii})$$

# 4. Isolating the Components of the Nominal Rate of Interest

The consumption-based CAPM model is motivated by the view that the real interest rate is driven by households' intertemporal consumption and savings decisions. It puts these decisions in a stochastic setting. The theoretical components of the nominal interest rate are dictated by the elements that enter into the theoretical CAPM pricing model. If the data suggest that the model is a good representation of household consumption and saving behaviour, then the relative magnitudes of the various components of the nominal interest rate can be inferred from the estimated model. Specifically, it allows us to identify and isolate the components of the equilibrium real rate and helps to obtain an estimate of it.

The consumption-based CAPM theory of the nominal interest rate as encapsulated in equation (vii) above identified these components, which are the following:

- i) The discount rate ( $\delta$ ), or (the inverse of) the rate of time preference;
- ii) Expected aggregate real consumption term,  $E(\Delta c)$ ;
- iii) The expected variance of consumption,  $Var(\Delta c)$ ;
- iv) Inflation expectations, i.e., the so-called Fisher effect,  $E(\Delta p)$ ;

- v) An inflation risk premium effect relating the nominal interest rate to the variability of inflation,  $Var(\Delta p)$ ; and
- vi) A covariance term,  $Cov(\Delta c, \Delta p)$ , capturing the risk to portfolios arising from the correlation between the business cycle and interest rates, with the latter assumed to be inversely related to changes in the price level.

The first component reflects the fact that the more heavily individuals discount the future (i.e., the higher is the discount rate), the greater is current consumption and the lower is current savings. Other things being equal, this drives up the real, and accordingly the nominal, rate of interest.

The second component is expected aggregate real consumption. For a given discount rate, the higher the expected growth rate of consumption the higher is future consumption relative to current consumption and the higher the interest rate has to be to prevent people transferring future consumption to the present, where consumption goods are in fixed supply. There is a natural corollary of this – when there is an incipient excess supply of current consumption and the lower the interest rate has to be relative to future consumption and the lower the interest rate has to be to encourage people to consume more now and less in the future. Expected consumption is captured by the one-period ahead forecast of consumption.

The third component captures uncertainty about future consumption. The more uncertain risk-averse individuals are about future consumption the more they will want to save now to insure themselves against this risk. Again, other things being equal, this higher level of savings drives down the real rate of interest and hence the actual nominal rate. Uncertainty about future consumption is measured by the variance of the one period ahead forecast for consumption.

The fourth component listed  $(E(\Delta p))$  is expected inflation. This is the standard variable used to test for the validity of the Fisher hypothesis as to whether nominal interest rates contain a full inflation premium. However, what bondholders are concerned about is the expected change in the purchasing power of money (i.e.,  $E(P_t / P_{t+1}))$  over the holding period of the bond and not the expected inflation rate over this period (i.e.,  $E(P_{t+1} / P_t))^2$ . In a world of uncertainty, according to Jensen's inequality, the expected value of one is not the inverse of the expected value of the other. According to Jensen's inequality, a mean-preserving spread in the inflation rate ( $Var(\Delta p)$ ) results in an increase in the expected purchasing power of money. Other things being equal, this results in an increase in the demand for bonds driving up bond prices and depressing bond yields. The relationship between the expected inflation and expected future purchasing power in a world of uncertainty can be written as follows:

$$\mathbf{E}(P_t / P_{t+1}) = \exp\left[-\mathbf{E}_t \left(\Delta P_{t+1}\right) + \frac{1}{2} \operatorname{Var}_t \left(\Delta P_{t+1}\right)\right]$$

The first term on the right hand side reflects the fact that bondholding households<sup>3</sup> expect to be compensated for any inflation, which they expect to occur over the holding period of the bond. If the current yield does not reward them for the inflation they expect to occur over the holding period of the bond, then they will sell off their bond holdings. This will have the effect of driving down the price of bonds and boosting the yield. Therefore, either an increase in expected inflation or a fall in the variance of future prices implies a decline in expected future purchasing power. This spills over into a reduction in demand for bonds, which inflates bond yields, which accounts for the positive and negative signs on these respective variables

 $<sup>^2</sup>$  This is a key distinction made by Evans and Wachtel (1992) whose model provides the core theoretical framework for this paper.

<sup>&</sup>lt;sup>3</sup> The introduction of collective investment schemes, in particular money market, bond and equity mutual funds, enabled retail investors to gain effective access to the securities' markets.

in the equation above<sup>4</sup>. The expected inflation rate is measured as the one period ahead forecast of inflation.

The fifth component noted above is designed to capture an inflation risk premium effect. This relates the nominal interest rate to the variability of inflation. If bondholding households are averse to risk, they will not welcome a situation in which there is doubt about the future purchasing power of their bond receipts at maturity. They will therefore expect to be compensated for this uncertainty by paying a lower price for the bond and thereby having a risk premium built into the nominal yield. The variance of the purchasing power of money one period ahead is used to measure this effect.

The final component is probably less familiar. It is another risk term, called the covariance risk. It captures the risk to portfolios arising from the correlation between the business cycle and interest rates. It is assumed that households would prefer to hold assets that would enable them to smooth consumption over time. They would accordingly want to hold assets that would yield a high return when income (and therefore consumption) is subject to cyclical downturn. This implies that they would want to hold assets that would co-vary negatively with consumption, i.e.,  $Cov(\Delta c, \Delta r) < 0$ . Since the expected real return on the bond portfolio varies negatively with the price level, households would therefore want to hold assets such that  $Cov(\Delta c, \Delta p) > 0$ . If, in fact, the asset is such that the first of these covariances is positive and the second negative, then households would expect to be compensated for this and, to be persuaded to hold the asset, would have to be rewarded with a risk premium (i.e., a covariance risk premium). This would then be built into the observed nominal rate of interest.

<sup>&</sup>lt;sup>4</sup> There have been other interpretations of the  $Var(\Delta P_{t+1})$  term. Friedman for example argued that inflation uncertainty erodes the efficiency of the market mechanism in allocating resources decreasing real output and shifting the aggregates supply curve to the left necessitating an increase in the real interest rate. This would suggest a positive sign on this variable. Others have argued that uncertainty about inflation reduces demand for investment and puts downward pressure on the real interest rate.

And, of course, this premium is in addition to the inflation premium and the consumption and inflation risk premiums already noted above, which are also built into the nominal rate.

A weighted combination of the first three terms noted above, the discount rate, expected real consumption and the variance of expected consumption comprise the equilibrium real rate of interest. This reflects the Fisherian theory of the real rate of interest as amended to take account of a stochastic setting and links the interest rate to consumer behaviour over time. It is clear from the derivation of this equation that, in the steady state, when there is no growth or variation in the level of consumption, the real equilibrium rate of interest is the same as the discount rate.

#### 5. Money and Monetary Uncertainty

The CCAPM framework does not allow for the type of nominal rigidities, which in practice would appear to be necessary for monetary policy to impact on real variables in the economy. The CCAPM model is implicitly couched in a barter framework and therefore abstracts from the fact that all transactions have to be mediated by money. Realism requires that the prevailing monetary system of exchange, and the ability of the central bank to influence this system by controlling the supply, or nowadays more likely the cost, of money transactions balances, be acknowledged. If individuals find themselves liquidity constrained due to a shortage of transactions balances, they will not be in a position to behave in accordance with the predictions of the consumption-based CAPM model.

If financial markets are relatively liquid, following a loosening of the stance of monetary policy, then the nominal rate of interest will be low relative to its fundamental determinants as represented by the consumption-based CAPM. The model must therefore be adjusted to capture this well-known Keynesian liquidity effect of monetary policy. To fully comprehend asset price determination, the liquidity preference theory of the interest rate must be

incorporated with classical loanable funds theory, which buttresses modern asset pricing models derived from utility theory. The observed rate of interest can differ from the natural rate depending on whether liquidity constraints are eased by an injection, or tightened by a withdrawal, of liquidity by the monetary authority.

Cash-in-advance constraints are incorporated into asset pricing models as a means of introducing liquidity effects. The Fuerst (1992) model is employed (which uses a methodology suggested by Lucas (1990) to establish the liquidity effect of monetary policy on the nominal rate of interest.

The economy ("family" in the Lucasian parable) maximises the expected discounted present value of uncertain future consumption subject to two cash-in-advance constraints; a cash balance growth constraint and market clearing conditions in five markets. The condition for the nominal bond rate of interest is:

$$(1+R_t) = \frac{\Lambda_t + U'(C)/P_t}{\beta \mathbb{E}\{U'(C_{t+1})/P_{t+1}\}}$$
(viii)

where:  $\Lambda_t = (\lambda_{2t} - \lambda_{1t})/M^s_t M^s_t$  is the beginning-of-time-period *t* per capita money stock.  $\lambda_{1t}$ and  $\lambda_{2t}$  denote the multipliers associated with the firm and shopper cash-in-advance constraints.  $\Lambda_t$  is called the liquidity effect. Where  $\Lambda_t$  equals zero, expression (viii) reduces to the standard Fisherian breakdown of the nominal rate of interest into a real rate (i.e., the intertemporal marginal rate of substitution) and an expected inflation premium.

Liquidity premiums arise when cash in one market is more valuable than cash in another market. The value of cash in the goods market is measured by  $\lambda_{1t}$  and the value of cash in the financial market is measured by  $\lambda_{2t}$ . It is expected that when liquidity effects are absent (i.e.,  $\lambda_{1t} = \lambda_{2t}$ ) the nominal rate should be determined by the Fisherian fundamentals. When the

financial market is relatively tight (i.e.,  $\lambda_{2t} > \lambda_{1t}$  and  $\Lambda_t > 0$ ) the nominal rate is high relative to the Fisherian fundamentals and vice versa when the financial market is relatively liquid.

The nominal rate of interest is affected by monetary policy actions that tighten or relax liquidity constraints via additions to or subtractions from M<sup>s</sup>. What is important for interest rate changes is exogenous changes to the M<sup>s</sup> rather than the existing level of stock as in the equation. This first order condition shows how money through a cash-in-advance constraint could modify the Fisher barter condition. Equation (viii) is therefore augmented by adding exogenous changes in the real money stock appearing as an additional argument to capture the liquidity effect of monetary policy on the nominal rate of interest.

The sign of the effect is unambiguously negative. A simple plot of the interest rate against the change in total bank reserves (which is used to proxy the liquidity effect) gives some visual confirmation of this (see Figure 1).



Figure 1: T-bill Rate and Change in Total Reserves, 1960:Q1-2005:Q2

There are now a large number of models that have been elaborated in order to incorporate money into asset pricing models. Some of them represent different strands of thinking in the literature. One popular strand of thinking argues that it is not money growth per se that has an effect on interest rates but rather monetary uncertainty (see, for example, Stulz (1986), Lee (1995), Evans and Lewis (1995) and Koedijk et al (1998)). The rationale, it appears, comes from Mascaro and Meltzer (1983) and argues that monetary uncertainty increases the asset demand for money driving up the real rate of interest.

The data could admit of both interpretations with *actual* money growth having a depressing effect on nominal and real interest rates in so-called "normal" times but monetary uncertainty boosting nominal and real rates in "abnormal" or turbulent times. The present paper allows for both effects to operate at least during some sub-periods of the sample.

# 6. Additional Considerations

Before going into a discussion of the details of the empirical results, some consideration has to be given to the maturity of the financial instrument whose yield is being used to capture the Fisher effect. The type of substitution possibilities that help the Fisher effect to hold may be inhibited in the short run by high transactions costs and the relative illiquidity of some assets. It is arguable then that the Fisher effect may only hold for yields on longer-maturity debt instruments. If an effect shows up for short rates it may only be because of an echo effect coming from long rates. But it is likely that this effect will not be systematic and therefore only a low or non-existent expected inflation premium is likely to show up in empirical testing. However, the actual correlation between longer-dated government securities and the three-month T-bill rate is very high and using the former makes no material difference to the results, see Figure 2. Moreover, the use of longer-dated securities gives rise

to overlapping observations and cumbersome estimation problems because the length of the investor horizon greatly exceeds the unit of time of the analysis.

Unless the effect of expected inflation on private bond market behaviour (as reflected in our model by the short-term interest rate) can be disentangled from the effect on monetary policy on the short-term interest rate, the two effects could be confounded in the empirical results. To get an unbiased Fisher effect it is important therefore to try and control for the monetary policy effect as precisely as possible. This is a challenging task, especially for a specification over the full sample period of the study, 1960Q1 to 2005Q2.



Figure 2: Government Securities, 1960:Q1 to 2005:Q2

The change in the Federal Reserve's monetary policy strategy toward a version of monetary targeting in October 1979 was also accompanied by a change in the operational framework of monetary policy. The monetarist strategy was implemented by a system of monetary base

control<sup>5</sup>. It was either this monetary policy strategy (fairly strict monetarism) and/or the way in which it was implemented (monetary base control as an operational framework) that resulted in a massive increase in interest rate volatility. Between October 1979 and September 1982 when the experiment was abandoned, the volatility of short-term interest rates increased four-fold relative to the pre-October 1979 period (see Figure 3 below). This suggests that this period was clearly an outlier, and should be treated as such. It is also clear that the heightened volatility was confined to this period. The experiment with monetarist policies was undoubtedly the dominant influence on the T-bill rate during this period. It does not therefore fit into the theoretical context of the present paper. In the regression results reported over the full sample period, this episode is therefore dummied out<sup>6</sup>.

The dummy variable (which takes on values of zero up to Q3 1979, values of one from Q4 1979 to Q3 1982, reverting to zero for the rest of the sample period) turns out to be very significant with a t-value of five. The dummy variable used here and that used by Tzavalis and Wickens (1996) could be capturing the effects of monetary uncertainty on the real and thus the nominal rate of interest. Koedijk et al examine the effect of monetary uncertainty explicitly. They proxy this by the conditional variance of money growth and include it as an argument in their empirical analysis. Replacing the dummy variable in our own analysis with an estimated measure of the conditional volatility of (M1) money stock growth indicates that the dummy variable is indeed, for the most part, capturing this volatility measure.

<sup>&</sup>lt;sup>5</sup> A monetarist strategy could also be implemented by the central bank guiding the evolution of the money stock via its control over the opportunity cost of holding money.

<sup>&</sup>lt;sup>6</sup> This is the approach also followed by Tzavalis and Wickens (1996).



Figure 3: Volatility of the Nominal Interest Rate, 1960:m1 to 2005:m6

Of course, it should be noted that conditional monetary volatility is not a substitute for money growth. The rationales for these variables are quite different. So, in contrast to the way money is treated in other similar exercises reported in the literature, both variables are included in our specification as potential simultaneous determinants of the real and nominal rates of interest.

## 7. Results for CCAPM with Monetary Policy Effects

Estimation of the generalised Fisher model requires the inclusion of the expectations, variances and covariance structure of consumption growth and the inflation rate, derived from autoregressive processes, see Annex 1. The estimated equations have good explanatory power and the derived conditional moments are employed as generated instruments in the estimation of the generalised Fisher model. Table A2-1 presents a summary of statistics for the components of the models to be estimated. All generated variables exhibit stationarity, with the exception of inflation, expected inflation and the interest rate variable. Estimations of inflation expectations follow a very similar pattern to inflation expectations extracted from

the Federal Reserve Bank of Philadelphia's 'Survey of Professional Forecasters' and the University of Michigan Survey Research Centre's 'Survey of Consumers' Inflation Expectations', see Figure A1-3 in Annex 1.

Other than those implicit in the generated variables that enter into the generalised Fisher relationship, there are no dynamics in any of the estimated equations. Despite this, the explanatory power of the estimated equations is quite high, ranging from 65 to 75 per cent. However, the absence of dynamics is probably responsible for the presence of residual autocorrelation in all of the estimated equations. We wished to avoid what would be a fairly ad hoc search for dynamics. We will address the issue of dynamics again below.

The CCAPM obtains fairly strong support from the data (see Table A1-3 in Annex 1). With one exception, all of the variables entering the model are signed according to theory. The Fisher coefficient is estimated to be 0.74 and is significantly different from both zero and one. Expected consumption is also significant with a coefficient of 0.16. The variance of expected inflation is correctly signed (a positive expected inflation risk premium) is only significant at the ten per cent level. The variance of expected consumption is incorrectly signed but statistically significant.

Figure 4 displays the covariance between the one-period ahead change in consumption and change in the price level. As has already been argued, risk-averse households would like this variable to be positive in which case they might be willing to accept a (small) discount in the yield on the bond



Figure 4: Covariance between the One-period Ahead Change in Consumption and Change in the Price Level, 1961:Q3 to 2005:Q2

Since it is mostly negative we would expect to see bondholders seeking a premium in the yield, i.e., a positive sign on the covariance term. This variable is therefore correctly signed according to theory but is not significant. But it should also be noted that the effect is very small. The average value of the covariance over the sample is less than half a percent of the average value of the T-bill rate, despite some significant outliers. It therefore makes very little contribution to explaining variation in the T-bill rate and could easily be dispensed with. Moreover, as can be seen from the figure, since about the early 1990s consumption and the price level have moved more or less independently of each other. This could be symptomatic of just how profound an effect financial market innovation has had in freeing aggregate consumption from *contemporaneous* correlation with other macroeconomic variables.

The way in which money enters the specification is designed to capture two different effects: a standard liquidity effect to be captured by (an instrumental variable for) the change in the monetary base expressed in real terms and, secondly, a monetary uncertainty effect assumed to be captured by the estimated conditional variance of the change in the M1 money stock. The first of these variables is included in the basic CCAPM model (recall result in Table A1-3 in Annex 1). It is correctly signed and is significant at the five and ten per cent levels.

We next include both money variables in the CCAPM specification (see "Estimation 1" result in Table A2-2 in Annex 2). Both are correctly signed with the total reserves (liquidity effect of monetary policy) being significant at the 1 per cent level and the monetary uncertainty variable significant at the 10 per cent level. There is therefore evidence of a conventional liquidity effect arising from monetary policy operations. There is also evidence that periods of monetary uncertainty cause investors to shy away from the T-bill market, which results in a small temporary premium being built into the real rate<sup>7</sup>.



Figure 5: Monetary Uncertainty, 1961:Q3 to 2005:Q2

Most of the episodes of monetary uncertainty seem to have been confined to certain periods (see Figure 5). They are clustered around the change in the monetary policy strategy and operational framework of the Fed in the 1979-1982 period and the events surrounding 9/11.

 $<sup>^{7}</sup>$  This is consistent with the findings of Koedijk et al (1998) for the US. Writing soon after the end of the monetarist experiment period, Holland (1984), concludes that the phenomenon most coincident with the high

The dummy variable for the former is significant and the monetary uncertainty variable remains significant, although at the ten per cent level. It is probably capturing other periods of uncertainty like the one that shows up in the data around 1985. Alternatively, even 'routine' monetary uncertainty may also be important in adding a risk premium. The significance of the two monetary variables is bought at the "expense" of the variances of expected consumption (still incorrectly signed but now significant) and inflation (now incorrectly signed and insignificant). Omitting the variance of expected inflation and the covariance terms from the specification makes no difference to the goodness of fit statistics (see "Estimations 2 and 3" in Table A2-2 in Annex 2).

#### 7.1 Results for CCAPM incorporating monetary policy exchange rates effects

Over the time period of the study, the globalisation of financial markets has ensured that any risk-adjusted discrepancy in interest rates across countries will be quickly arbitraged away. This means that exogenous changes in the yields on comparable asset denominated in foreign currencies could have a bearing on the T-bill rate. But since most of the variation in the deviation from uncovered parity will come from expected exchange rate change component of the foreign currency denominated return, we examine the effect of four such bilateral exchange rate changes in generalised Fisher relationship for the US economy.

The results are reported in Table A2-3 in Annex 2 of the paper. The results are mixed. Only two of the exchange rates are significant at 10 per cent significance level or higher, i.e., the \$/DM(euro) and the \$/£Sterling. The addition of these variables increases the explanatory power to the equation only fractionally, from 71 per cent to 73 per cent. Moreover, the inclusion of the exchange rate variable has resulted in the liquidity effect of monetary policy to fall to significance at only the 10 per cent level. The inclusion of these variables does

actual real rates in of the early 1980s was "...an increase in the variability of money growth, which increased

nothing to lessen the autocorrelation pattern of the estimated residuals. The estimated equilibrium rate from these results is lower than in the case of the pure CCAPM model.

# 7.2 Results from the Augmented Model

As already noted, the augmented model is the basic CCAPM model augmented by the inclusion of some macroeconomic variables. In the event, only three variables were considered, namely the price of oil, the federal government's fiscal surplus/deficit and industrial production. The results are reported in Table A2-4 in Annex 2 of the paper. Of the three macro variables, only the first two are significant, both at the one per cent level. Again, the additional explanatory power coming from these variables is only fractional, and increases from 73 per cent to 75 per cent. Moreover, a negative effect on the interest rate coming from the government budget surplus/deficit is difficult to rationalise. A negative impact effect of the price of oil on the nominal interest rate would not be unreasonable if the magnitude of the negative effect on aggregate demand, coming from the oil price shocks, were greater than the negative effect on aggregate supply. The third macro variable, industrial production, does not figure as an explanatory variable.

Our preferred specification is "Estimation 3" from this table, which excludes four insignificant variables from the first estimate in this table. Using this last specification to calculate a real equilibrium rate of interest yields fairly plausible results. This estimate, based on OLS results from the above equation, gives a mean value over the 40-year sample period of 2.26 per cent, which is in the same ballpark as other estimates for the US<sup>8</sup>. Figure 6 shows

economic uncertainty and the risk premium on interest rate."

<sup>&</sup>lt;sup>8</sup> A simple approach to the estimation of the equilibrium rate is to take the sample mean over a relatively long period when inflation is displaying no distinct upward or downward tends. Using this approach, Reifschneider and Williams (2000) report a value of about 3 per cent.

a plot of this equilibrium estimate against the corresponding actual real rate. The inflation rate over the same period is also plotted in the same figure<sup>9</sup>. The results are instructive.



Figure 6: Real Interest Rate Spread as an Indicator of Inflation, 1962:Q4 to 2005:Q1

<sup>&</sup>lt;sup>9</sup> In both cases the graph plots a six-month moving average of the data so as to allow the major developments in the equilibrium rate and in the stance of monetary policy to be seen more easily.

As noted already, one of the requirements of a sensible estimate of the natural rate is that it fluctuates much less that the actual real rate. This requirement is clearly fulfilled by our estimate of the natural rate. The difference between the two graphs is an estimate of the real interest rate spread. It seems to reveal six major episodes in the spread over the sample period. With only one exception, it seems to do quite a good job in accounting for inflation over the period of the sample. The first of these episodes starts with the increasingly loose stance of monetary policy throughout the 1960s and 1970s with the spread increasing to almost five and a half percentage points in late 1975. This is seen here as the factor predominantly responsible for the Great Inflation of the late 1960s and 1970s. It is also clear from the chart that the further the actual rate deviated from the equilibrium rate the more inflation accelerated.

The second episode starts with the Volker disinflation in the early 1980s. The Federal Reserve's restrictive monetary policies saw the actual real rate climb well above the equilibrium rate for a number of years (at one point in time to a maximum of over two percentage points). The upshot was a sharp disinflation, which saw inflation fall from over 11 per cent to below 4 per cent over approximately a three-year period. The third episode was the loosening that followed this, which, according to our estimates of the equilibrium rate, was overdone. The actual rate again fell below the equilibrium rate but only by about a percentage point. Nevertheless inflation accelerated again, albeit only modestly by about a percentage point. In this overall pattern, the next episode (i.e., episode 4 in the chart) is anomalous. Although the actual real rate fell the whole way to zero, while the equilibrium rate increased slightly, contemporaneous and subsequent inflation actually fell. One possible explanation for this anomaly could be that the bulk of the monetary pressure created by the

loose stance of monetary policy was deflected into asset markets and absorbed by asset price inflation rather than by inflation in consumer prices.

Episode 5 also fits the theory being propounded here. This episode lasted for most of the latter half of the 1990s and into the new century. It could be described as one of equilibrium with the real interest rate spread remaining close to zero for the full time period and inflation remaining more or less stable over the same period at about 2 per cent, as suggested by the real interest rate spread as an indicator.

The US economy is now in the middle of the final episode identified in the chart (i.e., episode 6) which began in late 2001 and which has been characterised by a very substantial widening of the real interest rate gap once again as the Fed endeavoured successfully to stave off deflation. These readings for the real interest rate spread would suggest that inflation should already have accelerated over the period 2002 and 2003 – in fact, it decelerated slightly. One possible explanation for this could be that this episode is like episode 4 where the monetary stress created by too loose a stance of monetary policy appears to have been dissipated in asset price rises rather than in consumer inflation. Indeed, developments in asset markets over the years 2003 and 2004 would suggest that it might have been prices of financial and real assets (defined to include the broad asset classes of equity, bonds, property and commodities) which all rose sharply over this time period that absorbed the monetary pressure coming from the real interest rate spread in the first instance.

Another possible explanation is that the lags between the real interest rate spread indicator and inflation has lengthened in the 1990s. This would suggest that the inflation predicted by the spread has yet to emerge. This explanation may amount to the same thing as the last one. The lengthening of the lag in transmission may be due to the effects of monetary policy following a more circuitous route to overall inflation via asset prices. The loose stance of monetary policy may be inflating asset prices, which could then eventually spill over into overall inflation via a wealth effect on aggregate consumption or via the effects on the investment component of GDP arising from a fall in the cost of market finance.

Despite the fact that we cannot reject the stationarity of the estimated residuals from all of these specifications, nevertheless a worrying feature of all of the results is the strong evidence of residual serial correlation. Augmenting the model by including extra variables not suggested by the basic CCAPM model, however, does nothing to resolve this problem.

We think it is likely that the remaining residual autocorrelation is due to the absence of dynamics in the estimated model. We have not undertaken a data-mining search for dynamics. However, there is one aspect of dynamics that could be taken on board. If the main factor causing the actual and equilibrium rates of interest to diverge is monetary policy, and if monetary policy is neutral with respect to all real prices in the long run, then the actual and equilibrium rates of interest should be cointegrated. This means that the actual real rate cannot wander too far away from the equilibrium real rate of interest. This, in turn, means that there is an error correction mechanism at work. The difference between the actual and equilibrium rates should cause the actual rate to adjust so as to eliminate the difference or, more likely, re-establish some small constant difference between the actual and equilibrium rates.

Taking an estimate of the equilibrium rate from our preferred specification (i.e., Estimation 3 in Table A2-4), and using a second-round estimate of the same equation in first difference form with this error correction term as an argument yields the result reported in Table A2-5. In this first-difference version of the equation there is no problem of first-order serial autocorrelation (according to the DW statistic). The error correction term is correctly signed and highly significant. It indicates a fairly rapid speed of adjustment of the actual rate to the

equilibrium rate. Movements in the actual rate eliminate on average about 21 per cent of the spread between the actual and equilibrium rates in a quarter.

# 8. Conclusions

Over recent years there has been a growing emphasis on interest rates as the dominant, if not exclusive, channel of transmission of monetary policy to the economy. The focus of practical central bankers has been increasingly on nominal interest rates in endeavouring to gauge the stance of monetary policy. However, nominal rates do not by themselves provide an anchor for monetary policy. If the central bank is focusing too closely on a particular value of the nominal interest rate, it may be inadvertently causing real rates to fall to a level that is too low for price stability.

There are two related dangers. Firstly, the central bank does not observe inflation expectations directly and therefore may not have an accurate picture of the level of the (ex ante) real interest rate corresponding to any given level of the nominal interest rate. It is, of course, the actual real rate that affects the level of activity in the economy. The second danger is an even greater source of concern. The central bank may not, in any case, have a good idea of where this actual real rate is relative to its corresponding equilibrium value, i.e., the natural rate. This is the rate that the theory suggests would ensure that monetary policy is neutral, in neither adding to nor subtracting from overall demand in the economy. This, in turn, suggests that the real interest rate spread (i.e., the actual minus the equilibrium real interest rate) should have a fairly high status as an indicator in the central bank's portfolio of monetary policy indicators. Our estimate of the real interest rate spread for the US economy, and a visual comparison of this with US inflation since the early 1970s, suggest that this variable could be useful as a measure of the degree of looseness or tightness in the Federal Reserve's monetary policy stance at any point in time.

The equilibrium real interest rate is not observed directly. It has to be estimated. This estimation is done here using an asset pricing model popular in the finance literature, specifically a consumption-based capital asset pricing model. The paper reports our results from this exercise.

The estimate of the equilibrium real rate for the US economy covering the period 1960 Q1 to 2005 Q2 hovers around the 2.5 per cent mark which is consistent with other estimates reported in the literature. The difference between this rate and the actual (ex ante) real rate tells a fairly plausible story about the evolution of inflation in the US economy over this period. We have identified about 6 salient episodes of inflation/disinflation over the period. A plot of the inflation rate against the estimated real interest rate spread gives a visual impression of the extent to which the spread can account for the evolution of inflation. It is obvious that both the sign and size of the spread can contribute substantially to explaining the history of inflation in the US over the sample period.

Only for one episode of the six does the indicator not perform as the theory predicts. This was for roughly the first half of the 1990s (specifically, 1991Q1 to 1996Q2) when a loose stance for the Federal Reserve's monetary policy, according to the real interest rate spread indicator, was accompanied by a fall in inflation of about 2 percentage points. One possible explanation for this is a once-off shift in liquidity preference. Another possible explanation is that the monetary tension created by the loose stance of monetary policy was absorbed in asset price inflation over this time period. This might suggest that the lag between the real interest rate spread and subsequent inflation may have altered with the growing participation of households and firms in financial markets.

The paper has not examined the relative forecasting power of this indicator relative to other much better known indicators such as the output gap or the deviation of unemployment from its NAIRU, a future research project. However, even if it turns out to be somewhat inferior in terms of forecasting power of future inflation, it could still be a better indicator from the central bank's point of view because it is immediately available and much less far removed from the monetary policy actions of the central bank than are these other indicators.

The fact that the real interest rate spread varies over time is not due just to the variation in the actual real interest rate. The equilibrium real rate of interest also changes over time but much less than the actual rate. It does so for two broad reasons. It varies with shifts in the demand for and supply of loanable funds coming from the private sector of the economy. It is also likely to have been subject to a trend evolution over time with the systematically changing structure of financial markets in the wake of financial market liberalisation. However, although the equilibrium real rate varies for these reasons, it does so only modestly relative to the actual real rate of interest, which, at any point in time, is heavily influenced by the stance of monetary policy.

Although the real interest rate spread seems to do a fairly good job of accounting for inflation over the sample period in the US, it may not constitute a sufficient statistic for the amount of inflationary/disinflationary/deflationary tensions in the economy generated by monetary policy actions. It may need to be supplemented by a real monetary quantity spread variable (some version of a real money gap) before a comprehensive picture of the medium-to-long run threat to price stability can be had.

#### References

Akaike, H. (1973). "Information Theory and the Extension of the Maximum Likelihood Principle." In 2<sup>nd</sup> International Symposium on Information Theory. Petrov, B.N., and F. Csaki (Eds).Budapest:Akademiai Kiado

Blanchard, O., and Lawrence H. Summers. (1984). "Perspectives on high world real interest rates." Brookings Papers on Economic Activity 2, 273-324.

Cochrane, J. (1994). "Discrete Time Empirical Finance." Unpublished Manuscript, University of Chicago.

Evans, M., and P. Wachtel. (1992). "Interpreting the Movements in Short-Tern Interest Rates." Journal of Business 65, 395-429.

Evans, M., and K. Lewis. (1995). "Do Expected Shifts in Inflation Affect Estimates of the Long-run Fisher Relation?" Journal of Finance 50, 225-253.

Fuerst, T.S. (1992). "Liquidity, Loanable Funds and Real Activity." Journal of Monetary Economics 29, 3-24.

Hetzel R. (2000). "The Taylor Rule: Is it a Useful Guide to Understanding Monetary Policy?" Economic Quarterly, Federal Reserve Bank of Richmond 86/2, Spring.

Holland, A.S. (1984). "Real interest rates: What accounts for their recent rise?" Federal Reserve Bank of St. Louis Review, December 1984.

Koedijk, K., Kool, C., and F. Nissen. (1998). "Real Interest Rates and Shifts in Macroeconomic Volatility." Journal of Empirical Finance 5, 241-261.

Lee, S.S. (1995). "Macroeconomic Sources of Time-Varying Risk Premia in the Term Structure of Interest Rates." Journal of Money Credit and Banking 27, 549-569.

Lucas, R.E. (1990). "Liquidity and Interest Rates." Journal of Economic Theory 50, 237-264.
Mascaro, A., and A. Meltzer. (1983). "Long and Short Term Interest Rates in a Risky World." Journal of Monetary Economics 12, 485-518.

Neiss, K., and E. Nelson. (2001). "The Real Interest Rate Gap as an Inflation Indicator." CEPR Discussion Papers 2848. C.E.P.R. Discussion Papers.

Reifschneider, D., and J. Williams. (2000). "Three Lessons for Monetary Policy in a Low-Inflation Era." Journal of Money Credit and Banking 32, 936-966.

Stulz, R. (1986). "Interest Rates and Monetary Policy Uncertainty." Journal of Monetary Economics 17, 331-347.

Schwarz, G. (1978). "Estimating the Dimension of a Model." Annals of Statistics 6, 461-464.

Tzavalis, E., and M. Wickens. (1996). "Forecasting Inflation from the Term Structure." Journal of Empirical Finance 3, 103-122.

Wickesell, K. (1936). Interest and Prices (translation of the 1898 edition by R.F. Kahn), London: Macmillian.

Woodford, M. (2003). Interest and Prices, Foundations of a Theory of Monetary Policy. Princeton, NJ. : Princeton University Press.

The generalised Fisher model requires the inclusion of expected inflation and expected consumption growth, and this annex explains the models used to generate the necessary variables. An autoregressive process is employed to obtain generated instruments for expected inflation and expected consumption. Akaike Information Criteria and Schwarz Bayesian Information Criteria tests were conducted to select the appropriate lag length for the consumption growth and inflation rate time series models. For the consumption and inflation models containing quarterly data, results of the tests suggest an autoregressive process of order two<sup>10</sup> is specified for the models as economic theory suggests that both consumption and inflation in the present period are related to lagged values of themselves with some disturbance.

The time-series models used to derive the estimates of the conditional moments of inflation and consumption growth are of the general form:

 $y_{t} = x_{t} f(B_{t}) + \varepsilon_{t}$ where;  $y_{t} = [\Delta c_{t}, \Delta p_{t}]'$   $B_{t} = [\beta_{1t}, \beta_{2t}, \beta_{3t}, \alpha_{1t}, \alpha_{2t}, \alpha_{3t}]'$   $x_{t} = \begin{bmatrix} 1, \ \Delta c_{t-1}, \ 0, \ 0\\ 0, \ 0, \ 1, \ \Delta p_{t-1} \end{bmatrix}$ 

Estimation of the consumption model  $\Delta c_t = \beta_{1t} + \beta_{2t} \Delta c_{t-1} + \beta_{3t} \Delta c_{t-2} + \varepsilon_t^c$ , produced the results presented in Table A1-1. Table A1-2 reports the results of the estimated inflation equation:  $\Delta p_t = \alpha_{1t} + \alpha_{2t} \Delta p_{t-1} + \alpha_{3t} \Delta p_{t-2} + \varepsilon_t^p$ 

 $<sup>^{10}</sup>$  The consumption and inflation models were estimated using a number of different lag lengths for both quarterly and monthly data, however the AR(2) processes were found to contain the most significant results for both models and yielded the highest explanatory power.

| Coefficient          | Estimates        |  |
|----------------------|------------------|--|
| $\beta_1$            | 0.46<br>(3.94)   |  |
| $oldsymbol{eta}_2$   | 1.20<br>(16.90)  |  |
| $oldsymbol{eta}_{3}$ | -0.35<br>(-4.91) |  |
| $R^2$                | 0.82             |  |
| DW                   | 2.06             |  |

Table A1-1: OLS estimates; Consumption growth model.

**Note:** The  $R^2$  refers to the goodness of fit. The DW refers to the Durbin-Watson statistic. The t-statistics are reported in the parentheses below the parameter estimates. The critical values for the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively.

Figure A1-1: Consumption growth model, 1961:Q3-2005:Q2



| Table A1-2: OLS estimates: Inflation rate m | odel. |
|---|-------|
|---|-------|

| Coefficient     | Quarterly data   |  |
|-----------------|------------------|--|
| $\alpha_{_{1}}$ | 0.14<br>(2.08)   |  |
| $lpha_2$        | 1.45<br>(21.80)  |  |
| $lpha_{_3}$     | -0.48<br>(-7.23) |  |
| $R^2$           | 0.97             |  |
| DW              | 2.13             |  |

**Note:** The  $R^2$  refers to the goodness of fit. The DW refers to the Durbin-Watson statistic. The t-statistics are reported in the parentheses below the parameter estimates. The critical values for the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively.



Figure A1-2: Inflation rate model, 1961:Q3-2005:Q2

Figures A1-1 and A1-2 display the actual and fitted values of the estimated consumption and inflation models. The estimated equations have good explanatory power and the derived conditional moments are employed as generated instruments in the estimation of the generalised Fisher model. Figure A1-3 displays the expected inflation rate as estimated against the Federal Reserve Bank of Philadelphia 'Survey of Professional Forecasters' and the University of Michigan Survey Research Centre's 'Survey of Consumers' inflation expectations'. The estimated expected inflation rate follows a similar pattern to the inflation expectations extracted from the surveys.





Estimation of the generalised Fisher equation requires the expectations of consumption growth and inflation rate and the variance-covariance structures estimated from the consumption and inflation models. In order to arrive at an estimable generalised Fisher equation,  $E_t \Delta c_{t+1}$ , is replaced by the fitted values from the estimated consumption model;  $E_t \Delta p_{t+1}$  is replaced by fitted values from the estimated inflation model. The conditional variance of consumption,  $Var_t(\Delta c_{t+1})$  is taken to be $(\varepsilon_{t+1}^c)^2$ ; the conditional variance of inflation,  $Var_t(\Delta p_{t+1})$  is  $(\varepsilon_{t+1}^p)^2$ ; the conditional covariance term is described as  $(\varepsilon_{t+1}^c, \varepsilon_{t+1}^p)$ , where the  $\varepsilon$  s are the estimated residuals from the consumption and inflation models.

Submission of these generated instruments into the generalised Fisher equation allows equation (vii) to be rewritten as:

$$i_{t} = \delta + \gamma_{1t} \hat{p}_{t+1} + \gamma_{2t} (\varepsilon_{t+1}^{p})^{2} + \gamma_{3t} (\varepsilon_{t+1}^{c}, \varepsilon_{t+1}^{p}) + \gamma_{4t} \hat{c}_{t+1} + \gamma_{5t} (\varepsilon_{t+1}^{c})^{2}$$
(vii)'

Table A1-3 presents the results from estimated equation (vii)', i.e., the specification of the generalised Fisher model incorporating exogenous changes in the money stock.

| Variable                         | Estimates |  |
|----------------------------------|-----------|--|
| Discount rate                    | 1.50      |  |
|                                  | (2.66)    |  |
| Expected consumption             | 0.16      |  |
|                                  | (1.49)    |  |
| Variance of expected consumption | 0.67      |  |
|                                  | (2.98)    |  |
| Expected inflation               | 0.74      |  |
|                                  | (11.06)   |  |
| Variance of expected inflation   | 0.60      |  |
|                                  | (1.66)    |  |
| Covariance                       | 0.21      |  |
|                                  | (0.51)    |  |
| Total reserves                   | -0.04     |  |
|                                  | (-2.11)   |  |
| $R^2$                            | 0.65      |  |
| DW                               | 0.33      |  |
| ADF                              | -3.90***  |  |

Table A1-3: OLS estimates; Generalised Fisher Liquidity Model

**Note:** The DW refers to the Durbin-Watson statistic. The R<sup>2</sup> refers to the goodness of fit. The t-statistics are reported in the parentheses below the parameter estimates. The critical values for the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively. The ADF refers to the augmented Dickey-Fuller statistic. The OLS residuals are used to construct the ADF statistic. The null hypothesis is that the series is I(1). \*\*\* denotes rejection at the 1% level, \*\* denotes rejection at the 5% level and \* denotes rejection at the 10% level.

### **Annex 2: Empirical Results**

Table A2-1 presents a summary of statistics for the components of the models to be estimated. From 1960: Q1 to 2005: Q2 the average interest rate was 5.58 per cent and the average inflation rate was 4.32 per cent over the period 1961Q1 to 2005Q2. All the variables reported in the table exhibit skewness and excess kurtosis in their distributions. At the 10 per cent significance level, the null hypothesis of a normal distribution is rejected for all of the variables. It is not possible to reject the null hypothesis of a unit root for the nominal interest rate, the inflation rate and expected inflation.

For reasons outlined in the main text (see sections 5 and 6), equation (vii)' is adjusted to include three additional arguments, namely a dummy variable and a variable relating to monetary uncertainty.

$$i_{t} = \delta + \gamma_{1t}MU + \gamma_{2t}D_{1} + \gamma_{3t}\hat{p}_{t+1} + \gamma_{4t}(\varepsilon_{t+1}^{p})^{2} + \gamma_{5t}(\varepsilon_{t+1}^{c}, \varepsilon_{t+1}^{p}) + \gamma_{6t}\hat{c}_{t+1} + \gamma_{7t}(\varepsilon_{t+1}^{c})^{2} - \gamma_{8t}\Delta m/p \quad (ix)$$

Estimation of equation (ix) reveals the results reported in Table A2-2. "Estimation 1" refers to an assessment of equation (ix) in its complete form. The insignificant result for the variance of expected consumption implies that this variable does not significantly contribute to the risk premium. Hence the variance of expected consumption is eliminated from the assessment due to its insignificant outcome and the equation is re-estimated. "Estimation 2" reveals that the coefficient for the variance of expected inflation remains negative and subtracts from the nominal interest rate. It remains insignificant in "Estimation 2" and is thus removed from the evaluation of "Estimation 3".

## **Table A2-1: Summary statistics**

|                      | i        | $\Delta C$ | $\Delta P$ | $\Delta M$ | Var $\Delta M1$ | $\Delta S^{Ger}$ | $\Delta S^{Jap}$ | $\Delta S^{UK}$ | $\Delta S^{Can}$    |
|----------------------|----------|------------|------------|------------|-----------------|------------------|------------------|-----------------|---------------------|
| Mean                 | 5.58     | 3.17       | 4.32       | -0.74      | 1.99            | -2.15            | -0.60            | 0.53            | 1.15                |
| Standard Error       | 2.77     | 1.64       | 2.65       | 6.80       | 3.92            | 18.30            | 16.56            | 15.26           | 6.46                |
| Minimum              | 0.92     | -2.43      | 0.65       | -15.69     | 0.002           | -90.32           | -45.75           | -50.91          | -26.31              |
| Maximum              | 15.05    | 7.40       | 12.01      | 20.38      | 36.99           | 64.36            | 56.72            | 61.25           | 21.58               |
| t-Statistic (mean=0) | 27.24*** | 25.76***   | 21.70***   | -1.44      | 6.74***         | -1.58*           | -0.49            | 0.47            | 2.41 <sup>abc</sup> |
| Skewness             | 1.01     | -0.47      | 1.17       | 0.64       | 5.10            | 0.14             | 0.18             | 0.38            | -0.61               |
| Kurtosis             | 1.53     | 0.88       | 0.71       | 0.60       | 37.48           | 3.13             | 0.79             | 2.41            | 3.20                |
| Jarque-Bera          | 48.81*** | 12.22***   | 44.31***   | 14.68***   | 11067.63***     | 75.08***         | 5.75*            | 48.45***        | 89.21***            |
| Dickey-Fuller        | -1.86    | -3.17**    | -1.43      | -3.23**    | -11.38***       | -10.47***        | -10.13***        | -10.77***       | -9.83***            |

\*\*\*denotes rejection at the 1% level, \*\* denotes rejection at the 5% level and \* denotes rejection at the 10% level. The relevant critical values of the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively. The relevant critical values for the chi-square distribution with two degrees of freedom are 9.21, 5.991 and 4.605 for 1%, 5% and 10% levels of significance respectively. The relevant critical values for the Dickey-Fuller unit root test are -3.46, -2.88, and -2.57 for 1%, 5% and 10% levels of significance respectively. The relevant critical values for the Dickey-Fuller unit root test are -3.46, -2.88, and -2.57 for 1%, 5% and 10% levels of significance respectively. Skewness is defined as N<sup>2</sup>/(N-1)(N-2)\*m<sub>3</sub>/s<sup>3</sup>. Kurtosis is defined as N<sup>2</sup>/(N-1)(N-2)(N-3)\*(N+1)m<sub>4</sub>-3(N-1)m<sup>2</sup>/s<sup>4</sup>. A value for skewness of zero and a value of three for kurtosis is necessary for a normal distribution. The Jarque-Bera test statistic for normality is based upon the measures of skewness and kurtosis and is defined as N[(Ku<sup>2</sup>/24)+(Sk<sup>2</sup>/6)], where Ku denotes kurtosis and Sk denotes skewness.

# Table A2-1: Summary statistics cont.

.

|                      | $\Delta Oil$ | $\Delta$ Budget | $\Delta i p$ | $\hat{p}$ | ĉ        | $\left( \boldsymbol{\varepsilon}^{p} \right)^{2}$ | $(\boldsymbol{\varepsilon}^{c})^{2}$ | $\left( \boldsymbol{\mathcal{E}}^{c}, \boldsymbol{\mathcal{E}}^{p}  ight)$ |
|----------------------|--------------|-----------------|--------------|-----------|----------|---|--------------------------------------|--|
|                      |              |                 |              |           |          |   |                                      |  |
| Mean                 | 5.86         | 14.77           | -0.87        | 4.35      | 3.18     | 0.20  | 0.49                                 | -0.16  |
| Standard Error       | 32.27        | 87.6            | 5.80         | 2.61      | 1.50     | 0.49  | 0.94                                 | 0.57   |
| Minimum              | -50.95       | -90.79          | -19.62       | 0.51      | -2.24    | 0.000004  | 0.00003                              | -6.19  |
| Maximum              | 158.37       | 709.75          | 11.58        | 12.11     | 7.21     | 4.13  | 9.28                                 | 0.84   |
| t-Statistic (mean=0) | 2.42***      | 2.25**          | -2.00**      | 22.15***  | 28.16*** | 5.58***   | 6.89***                              | -3.71***   |
| Skewness             | 2.38         | 5.70            | -0.85        | 1.19      | -0.43    | 5.66  | 5.65                                 | -7.37  |
| Kurtosis             | 7.52         | 37.42           | 0.76         | 0.82      | 1.00     | 38.72   | 45.47                                | 74.49  |
| Jarque-Bera          | 586.67***    | 11349.38***     | 25.88***     | 46.71***  | 12.88*** | 11932.79***                                       | 16099.97***                          | 42282.03***  |
| Dickey-Fuller        | -4.73***     | -6.72***        | -2.76*       | -1.92     | -4.09*** | -8.60***  | -11.64***                            | -12.71***  |

\*\*\* denotes rejection at the 1% level, \*\* denotes rejection at the 5% level and \* denotes rejection at the 10% level. The relevant critical values of the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively. The relevant critical values for the chi-square distribution with two degrees of freedom are 9.21, 5.991 and 4.605 for 1%, 5% and 10% levels of significance respectively. The relevant critical values for the Dickey-Fuller unit root test are -3.46, -2.88, and -2.57 for 1%, 5% and 10% levels of significance respectively. The relevant critical values for the Dickey-Fuller unit root test are -3.46, -2.88, and -2.57 for 1%, 5% and 10% levels of significance respectively. Skewness is defined as N<sup>2</sup>/(N-1)(N-2)\*m<sub>3</sub>/s<sup>3</sup>. Kurtosis is defined as N<sup>2</sup>/(N-1)(N-2)(N-3)\*(N+1)m<sub>4</sub>-3(N-1)m<sup>2</sup>/s<sup>4</sup>. A value for skewness of zero and a value of three for kurtosis is necessary for a normal distribution. The Jarque-Bera test statistic for normality is based upon the measures of skewness and kurtosis and is defined as N[(Ku<sup>2</sup>/24)+(Sk<sup>2</sup>/6)], where Ku denotes kurtosis and Sk denotes skewness.

| Variable                         | Estimation 1     | Estimation 2   | Estimation 3     |
|----------------------------------|------------------|----------------|------------------|
|                                  |                  |                |                  |
| Discount rate                    | 0.84<br>(1.56)   | 0.91<br>(1.72) | 0.92<br>(1.76)   |
| Expected Consumption             | 0.40             | 0.39           | 0.39             |
| Variance of expected consumption | 0.41<br>(1.95)   | 0.42<br>(1.99) | 0.40<br>(2.86)   |
| Expected inflation               | 0.69<br>(10.91)  | 0.68 (11.12)   | 0.68<br>(11.19)  |
| Variance of expected inflation   | -0.26<br>(-0.64) | _              | _                |
| Covariance                       | -0.09<br>(-0.22) | 0.05<br>(0.14) | -                |
| Total reserves                   | -0.05<br>(-2.54) | -0.05          | -0.05<br>(-2.49) |
| Monetary uncertainty             | 0.07             | 0.06           | 0.06 (1.80)      |
| Dummy 1                          | 3.38             | 3.35           | 3.35             |
| $R^2$                            | 0.71             | 0.71           | 0.71             |
| DW                               | 0.36             | 0.34           | 0.34             |
| ADF                              | -4.16***         | -4.00***       | -4.00***         |

 Table A2-2: OLS estimates of the generalised liquidity Fisher model.

**Note:** The DW refers to the Durbin-Watson statistic. The R<sup>2</sup> refers to the goodness of fit. The t-statistics are reported in the parentheses below the parameter estimates. The critical values for the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively. The ADF refers to the augmented Dickey-Fuller statistic. The OLS residuals are used to construct the ADF statistic. The null hypothesis is that the series is I(1). \*\*\* denotes rejection at the 1% level, \*\* denotes rejection at the 5% level and \* denotes rejection at the 10% level.

The Durbin-Watson statistic in "Estimation 3" of 0.34 suggests serial autocorrelation. Further consideration of the form of the generalised Fisher liquidity model proposes that a relevant variable has been omitted from the specified model. Section 7.1 in the main text explains the rationale for including exchange rates into the CCAPM with monetary policy effects. The Fisher hypothesis states that the nominal rate of interest moves one for one with inflation, while the real rate of interest is a constant, i.e.,  $i_t = i_r + \pi$ , where  $i_n$  is the nominal rate of interest,  $i_r$  is the real rate of interest and  $\pi$  is the expected inflation rate. The Fisher equation responds to shifts in the exchange rate as these movements affect inflation calculated by the

consumer price index. Thus it would seem plausible to include exchange rates into the generalised Fisher liquidity model.

Consequently equation (ix) is reformulated to include the change in the exchange rate for four of the US's main trading partners, namely Germany, Japan, UK and Canada.

$$i_{t} = \delta + \gamma_{1t} M U + \gamma_{2t} D_{1} + \gamma_{3t} \hat{p}_{t+1} + \gamma_{4t} (\varepsilon_{t+1}^{p})^{2} + \gamma_{5t} (\varepsilon_{t+1}^{c}, \varepsilon_{t+1}^{p}) + \gamma_{6t} \hat{c}_{t+1} + \gamma_{7t} (\varepsilon_{t+1}^{c})^{2} - \gamma_{8t} \Delta m / p + \gamma_{9t} \Delta s_{t+1}^{Ger} + \gamma_{10t} \Delta s_{t+1}^{Jap} + \gamma_{11t} \Delta s_{t+1}^{UK} + \gamma_{12t} \Delta s_{t+1}^{Can}$$
(x)

A summary of statistics for the four exchange rates is contained within Table A2-1. Table A2-3 presents the results from estimated equation (x). "Estimation 2" finds the change in the exchange rate US/Canada to be insignificant. "Estimation 3" omits the change in the exchange rate US/Canada and estimation reveals the change in the exchange rate US/Canada remains insignificant and is hence removed from "Estimation 3".

| Variable                         | Estimation 1 | Estimation 2 | Estimation 3 |
|----------------------------------|--------------|--------------|--------------|
| Discount rate                    | 0.75         | 0.81         | 0.80         |
|                                  | (1.40)       | (1.57)       | (1.55)       |
| Expected Consumption             | 0.44         | 0.44         | 0.44         |
|                                  | (4.13)       | (4.12)       | (4.17)       |
| Variance of expected consumption | 0.36         | 0.35         | 0.36         |
|                                  | (1.74)       | (2.57)       | (2.58)       |
| Expected inflation               | 0.69         | 0.68         | 0.69         |
|                                  | (11.08)      | (11.37)      | (11.52)      |
| Variance of expected inflation   | -0.23        | -            | -            |
|                                  | (-0.56)      |              |              |
| Covariance                       | -0.08        | -            | -            |
|                                  | (-0.21)      |              |              |
| Total reserves                   | -0.03        | -0.03        | -0.03        |
|                                  | (-1.76)      | (-1.70)      | (-1.69)      |
| Monetary uncertainty             | 0.07         | 0.06         | 0.05         |
|                                  | (1.73)       | (1.71)       | (1.70)       |
| Dummy 1                          | 3.30         | 3.28         | 3.28         |
| 2 4                              | (5.13)       | (5.14)       | (5.16)       |
| Exchange rate: US/Germany        | 0.02         | 0.02         | 0.02         |
| Exemange rate: 00/00rmany        | (2.19)       | (2.22)       | (2.31)       |
| Exchange rate: US/Japan          | 0.01         | 0.01         | 0.01         |
| Exchange rate. 05/Japan          |              |              |              |
|                                  | (1.35)       | (1.39)       | (1.40)       |

 Table A2-3: OLS estimates of the generalised liquidity Fisher model incorporating exchange rates.

| Exchange rate: US/UK     | 0.02     | 0.02     | 0.02     |
|--------------------------|----------|----------|----------|
|                          | (1.92)   | (1.98)   | (1.99)   |
| Exchange rate: US/Canada | 0.01     | 0.01     | -        |
| -                        | (0.38)   | (0.32)   |          |
| $R^2$                    | 0.73     | 0.73     | 0.73     |
| DW                       | 0.45     | 0.43     | 0.43     |
| ADF                      | -4.69*** | -4.57*** | -4.56*** |

**Note:** The R<sup>2</sup> refers to the goodness of fit. The DW refers to the Durbin-Watson statistic. The tstatistics are reported in the parentheses below the parameter estimates. The critical values for the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively. The ADF refers to the augmented Dickey-Fuller statistic. The OLS residuals are used to construct the ADF statistic. The null hypothesis is that the series is I(1). \*\*\*denotes rejection at the 1% level, \*\*denotes rejection at the 5% level and \* denotes rejection at the 10% level.

For the generalised liquidity Fisher model including exchange rates, the statistically significant variables are correctly signed. The negative covariance term translates to a risk premium in the nominal interest rate, when the real return on an asset is moving in the wrong direction for risk adverse consumers. Risk adverse consumers tend to avoid holding assets, which give random variations in their consumption patterns. This arises from the tendency of risk adverse consumers to demand a premium on the risk free rate of interest in order to be compensated for holding an asset, which yields high returns when consumption is high and low returns when consumption is low. The estimated Fisher coefficient says that just over half of expected inflation is translated into the nominal interest rate. The proportion of the variation in the dependent variable,  $i_r$ , attributable to the linear combination of the regressors is 73 per cent. Positive serial autocorrelation remains strong within the model despite the inclusion of previously omitted relevant variables.

The rationale for an augmented model, i.e., the consumption based capital asset pricing model including exchange rates plus other macroeconomic variables is outlined in the main text (see section 2 of the main text). The macroeconomic variables considered for inclusion in the augmented model are the price of energy, the budget deficit/surplus and an index of

industrial production. Equation (x) is supplemented to include the chosen macroeconomic variables.

$$i_{t} = \delta + \gamma_{1t}MU + \gamma_{2t}D_{1} + \gamma_{3t}\hat{p}_{t+1} + \gamma_{4t}(\varepsilon_{t+1}^{p})^{2} + \gamma_{5t}(\varepsilon_{t+1}^{c}, \varepsilon_{t+1}^{p}) + \gamma_{6t}\hat{c}_{t+1} + \gamma_{7t}(\varepsilon_{t+1}^{c})^{2} - \gamma_{8t}\Delta m / p + \gamma_{9t}\Delta s_{t+1}^{Ger} + \gamma_{10t}\Delta s_{t+1}^{Jap} + \gamma_{11t}\Delta s_{t+1}^{UK} + \gamma_{12t}\Delta s_{t+1}^{Can} + \gamma_{13t}\Delta Oil + \gamma_{14t}\Delta Budget + \gamma_{15t}\Delta IP$$
(xi)

Results obtained from the estimation of equation (xi), excluding previously insignificant variables, are presented in Table A2-4. Analysis of "Estimation 2" led to the removal of the insignificant variable, industrial production, in "Estimation 3".

Table A2-4: OLS estimates of the generalised liquidity Fisher modelincorporating other macroeconomic variables.

| Variable                          | Estimation 1 | Estimation 2 | Estimation 3 |
|-----------------------------------|--------------|--------------|--------------|
| Discount rate                     | 1.18         | 1.26         | 1.27         |
| Discount rate                     | (2.22)       | (2.44)       | (2.45)       |
| Expected Consumption              | 0.30         | 0.30         | 0.32         |
|                                   | (2.62)       | (2.65)       | (3.01)       |
| Variance of expected consumption  | 0.33         | 0.37         | 0.37         |
|                                   | (1.62)       | (2.74)       | (2.76)       |
| Expected inflation                | 0.73         | 0.71         | 0.69         |
| •                                 | (2.22)       | (10.18)      | (11.87)      |
| Variance of expected inflation    | -0.39        | -            | -            |
| -                                 | (-1.00)      |              |              |
| Covariance                        | -0.26        | -            | -            |
|                                   | (-0.67)      |              |              |
| Fotal reserves                    | -0.04        | -0.04        | -0.04        |
|                                   | (-2.06)      | (-1.98)      | (-2.02)      |
| Monetary uncertainty              | 0.08         | 0.06         | -            |
| 5                                 | (2.04)       | (1.84)       |              |
| Dummy 1                           | 3.02         | 2.99         | 2.98         |
| 2                                 | (4.79)       | (4.80)       | (4.81)       |
| Exchange rate: US/Germany         | 0.02         | 0.02         | 0.02         |
| 5                                 | (2.25)       | (2.25)       | (2.24)       |
| Exchange rate: US/Japan           | 0.01         | 0.01         | 0.01         |
|                                   | (1.68)       | (1.75)       | (1.80)       |
| Exchange rate: US/UK              | 0.02         | 0.02         | 0.02         |
| 0                                 | (2.18)       | (2.26)       | (2.31)       |
| Exchange rate: US/Canada          | -0.004       | _            | -            |
| 5                                 | (-0.24)      |              |              |
| Dil price                         | -0.01        | -0.01        | -0.01        |
|                                   | (-2.81)      | (-2.79)      | (-2.82)      |
| Government budget deficit/surplus | -0.004       | -0.003       | -0.004       |
|                                   | (-2.39)      | (-2.41)      | (-2.84)      |
| ndustrial production              | 0.02         | 0.01         |              |
| hadden production                 | (0.61)       | (0.42)       |              |

| $R^2$ | 0.75     | 0.75     | 0.75     |
|-------|----------|----------|----------|
| DW    | 0.58     | 0.54     | 0.55     |
| ADF   | -5.42*** | -5.21*** | -5.27*** |

**Note:** The R<sup>2</sup> refers to the goodness of fit. The DW refers to the Durbin-Watson statistic. The tstatistics are reported in the parentheses below the parameter estimates. The critical values for the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively. The ADF refers to the augmented Dickey-Fuller statistic. The OLS residuals are used to construct the ADF statistic. The null hypothesis is that the series is I(1). \*\*\*\*denotes rejection at the 1% level, \*\* denotes rejection at the 5% level and \* denotes rejection at the 10% level.

| Variable                          | Estimation |  |
|-----------------------------------|------------|--|
|                                   | -0.63      |  |
| Discount rate                     | (-2.50)    |  |
| Error correction term             | -0.21      |  |
| Enor correction term              | (-6.00)    |  |
| Expected Consumption              | 0.14       |  |
| 1 1                               | (2.76)     |  |
| Variance of expected consumption  | 0.10       |  |
|                                   | (1.48)     |  |
| Expected inflation                | 0.20       |  |
|                                   | (5.44)     |  |
| Total reserves                    | -0.003     |  |
|                                   | (-0.30)    |  |
| Monetary uncertainty              | -0.01      |  |
|                                   | (-0.79)    |  |
| Dummy 1                           | 0.86       |  |
|                                   | (2.67)     |  |
| Exchange rate: US/Germany         | 0.01       |  |
|                                   | (1.65)     |  |
| Exchange rate: US/Japan           | 0.005      |  |
|                                   | (1.19)     |  |
| Exchange rate: US/UK              | 0.01       |  |
| 0.1                               | (1.23)     |  |
| Oil price                         | -0.001     |  |
|                                   | (-0.31)    |  |
| Government budget deficit/surplus | -0.002     |  |
|                                   | (-2.73)    |  |
| $R^2$                             | 0.29       |  |
| DW                                | 1.72       |  |
| ADF                               | -11.40***  |  |

### Table A2-5: Error correction model

**Note:** The R<sup>2</sup> refers to the goodness of fit. The DW refers to the Durbin-Watson statistic. The t-statistics are reported in the parentheses below the parameter estimates. The critical values for the t-distribution are 2.326, 1.96 and 1.645 for 1%, 5% and 10% levels of significance respectively. The ADF refers to the augmented Dickey-Fuller statistic. The OLS residuals are used to construct the ADF statistic. The null hypothesis is that the series is I(1). \*\*\*denotes rejection at the 1% level, \*\* denotes rejection at the 5% level and \* denotes rejection at the 10% level.

### **Annex 3: Data Sources and Methods Data definitions**

The performance of the generalised Fisher equation is assessed using quarterly data for the period 1960 to 2005. The chosen measure of the short-term nominal interest rate is the 3-month Treasury bill (secondary market rate), sourced from the Board of Governors of the Federal Reserve's Statistical Release H.15. The consumption data are the US Department of Commerce: Bureau of Economic Analysis's seasonally adjusted data on real consumption of nondurables and services.

The measure of inflation is based on the Consumer Price Index (CPI) less energy for all urban consumers. The rationale for the choice of this index, the CPI less energy, is that oil price shocks are frequently cited as being an underlying force to increases in the general price level. The data series for the CPI were obtained from the US Department of Labour: Bureau of Labour Statistics. The data series for money stock is compiled from the Board of Governors' total reserves adjusted for changes in reserve requirements, sourced from the Board of Governors of the Federal Reserve's Statistical Release H.3. Where data were available for monthly frequencies only, averages over three-month periods were calculated to obtain data series of a quarterly frequency.

The inflation rate, consumption growth and the change in reserve money are calculated using the formula:

$$x_t = [(x_t - x_{t-n}) / x_{t-n}] x 100$$

where n=4 for quarterly data.

The variable representing monetary uncertainty is derived from M1 money stock, sourced from the Board of Governors of the Federal Reserve System, Release H.6 Money Stock Measures. The exchange rate data are obtained from the Board of Governors of the Federal

Reserve System, Release H10. Foreign Exchange Rates. The measure chosen for the price of energy is the spot oil price: West Texas intermediate, obtained from the Federal Reserve Bank of St. Louis who have reprinted the data with permission from the Dow Jones Energy Service. The government budget surplus/deficit data are obtained from the US Department of Commerce: Bureau of Economic Analysis, and the industrial production data are sourced from the Board of Governors of the Federal Reserve System, release G.17 Industrial Production and Capacity Utilization.