3/RT/03 July 2003

CENTRAL BANK AND FINANCIAL SERVICES AUTHORITY OF IRELAND
Research Technical Paper

Dynamic Factor Demands in a Changing Economy: An Irish Application

by

Kieran McQuinn*

Economic Analysis and Research Department
Central Bank of Ireland
P.O. Box 559, Dame Street
Dublin 2
Ireland
http://www.centralbank.ie/research.asp

*The views expressed in this paper are the personal responsibility of the author. They are not necessarily held either by the Central Bank of Ireland or the ESCB. E-mail: kmcquinn@centralbank.ie. The author would like to thank all those who participated at a seminar in the Central Bank, in particular, Maurice McGuire, John Frain and Karl Whelan. Any mistakes

Abstract

In this paper a model of dynamic factor demands is presented for the Irish economy. Total costs, labour and capital are modelled on a two-stage basis. Firstly, a static, long-run cost function is specified which allows for the derivation of expressions for optimal labour and capital demand. This function is assumed to be of the flexible, translog form and thus more general than the generic Cobb-Douglas application. In the second stage, a dynamic cost function is specified which nests the long-run static approach. Growth rates in factor shares are derived from the dynamic approach and the rate of adjustment of input use to factor price changes is examined through the use of short and long-run elasticities.

are the sole reponsibility of the author.

1 Introduction

Capturing the appropriate dynamics and structural representation of the supply side of an economy is an important and complex exercise. Key economic indicators such as output gaps and technical productivity are often conditional on the underlying model assumed to characterise the production behaviour of firms in the economy. For instance, the output gap is sometimes measured as the deviation of actual output from an output level predicted from an estimated production function at the potential level of employment.

Applied analysis of factor demands necessitates the assumption of a functional form which approximates the technology of the firms operating in a specific economy. A production function or a dual cost function is specified, and factor demands are subsequently derived and estimated. Factor input price elasticities along with elasticities of substitution can then be estimated and the parameters of the underlying production function retrieved. In general, most empirical investigations are characterised by two traits

(1) the underlying functional form of the production/cost function employed is usually relatively restrictive,

(2) factor demands are estimated within a static rather than a dynamic context.

As noted by Gundlach (2001), most structural models of productivity growth tend to avail of the Cobb Douglas functional form (see Slevin (2001) and Allen and Mestre (1997) as recent examples). While tractable, the Cobb-Douglas is generally regarded as being quite restrictive compared to other functional forms. The primary restriction associated with the Cobb-Douglas is the imposition of constant factor shares due to the imposed unit elasticity of substitution between factor inputs. The adoption of a flexible, functional form such as the translog or normalised quadratic, on the other hand imposes no such restrictions on the elasticity of substitution. By definition, a flexible, functional form is a form which has enough parameters to capture the elasticity of substitution without imposing prior restrictions on the particular relationship in question.[1]

[1] For a comprehensive treatment of this issue see Chambers (1988) amongst others.

Static analysis of factor demands implies an instantaneous adjustment by firms in their utilisation of labour and capital to price shocks in the production process. In the case of capital the assumption is particularly unrealistic. For instance, if firms were able to adjust their capital stock everytime the interest rate changed, it could conceivably lead to either an infinite accumulation or scrapping of capital. Therefore, a more realistic approach is to assume that for a given factor price change, firms incurr adjustment costs in any alterations in their levels of factor inputs. To ignore these costs, as is the case in static analysis, is to ignore a significant component of the costs facing a firm in an expansionary phase of its production cycle.[2]

Therefore, the approach adopted in this paper is to apply the dynamic flexible form cost function and related dynamic factor shares model advanced in Allen and Urga (1995) and Allen and Urga (1999) to quarterly national income accounts data (1981:1-1999:4) of the Irish economy. The Allen and Urga (1999) model builds on an earlier approach by Anderson and Blundell (1982) by specifying the underlying objective function from which the factor demand functions are derived. The original Anderson and Blundell (1982) approach had just presented the dynamic factor demands. The adoption of this generalised model will enable the generation of not just the elasticity of substitution between capital and labour in the Irish economy between 1981 and 1999, but also both long and short run elasticities of demand for the factor inputs. In addition, the use of a more general flexible functional form as an approximate of the level of technology in the economy should permit an enhanced specification of key macroeconomic indicators such as the output gap and total factor productivity (TFP) as measured by the Solow Residual.

The paper is laid out as follows; the next section outlines a long-run translog cost function of the Irish economy, this is followed by an introduction to the dynamic cost function as proposed by Allen and Urga (1999). Section 4 presents the results of the estimations and a final section offers some conclusions.

[2]Of course the relative inflexibility of a factor input can be accomodated within a static dual framework through the adoption of a restricted or short-run cost function.

2 A Static Model of Irish Factor Demands

The model presented in this section is similar to that presented in Hall and Nixon (1999). The supply-side of the Irish economy is treated as a representative firm operating under conditions of imperfect competition with two factor inputs - labour and capital. Factor prices for both labour and capital are treated as given and optimal levels for both inputs are determined for a given state of technology. A disembodied level of technical progress is also assumed.

The following list of variables are used in the firm's decision making process:

$$\begin{aligned}
Q &= \text{aggregate output.} \\
C &= \text{total aggregate costs.} \\
L &= \text{aggregate labour.} \\
K &= \text{aggregate capital.} \\
P_L &= \text{price of aggregate labour L.} \\
P_K &= \text{price of aggregate capital (cost of capital) K.} \\
S_L &= \text{share of total costs attributable to labour.} \\
S_K &= \text{share of total costs attributable to capital.} \\
T &= \text{technical progress.}
\end{aligned}$$

Aggregating across all firms within the domestic economy, the objective function for the imperfectly competitive firm may be summarised as

$$\mathbf{min}\ \ \mathbf{C}_t = C_t\left(P_{Kt}, P_{Lt}, Q_t, T\right) \tag{1}$$

Factor Demands for labour and capital are obtained by applying Shephard's lemma to the cost function. Given the objective function, the next issue is to approximate the underlying technology of the producer using a flexible functional form. A functional form is flexible when it has enough parameters to examine the different relationships between the factor inputs such as the elasticity of substitution without imposing some prior restriction on the relationships in question. For instance, although the Cobb Douglas function is one of the best known and popular functional forms, it cannot be used to investigate the elasticity of substitution between different inputs as it imposes an elasticity of substitution of 1. The functional form adopted here is the translog form. Consequently, the log (ln) of total costs (C) may be approximated as

$$
\begin{aligned}
lnC = \omega_0 + \omega_Q\ lnQ_t + \omega_T\ T + \sum_{i=1}^{2} \omega_i\ lnP_i + \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \omega_{ij}\ lnP_i\ lnP_j \\
+ \sum_{i=1}^{2} \omega_{iQ}\ lnQ\ lnP_i + \sum_{i=1}^{2} \omega_{iT}\ T\ lnP_i + \omega_{QT}\ T\ lnQ \\
+ \frac{1}{2}\omega_{QQ}\ (lnQ)^2 + \frac{1}{2}\omega_{TT}\ (T)^2
\end{aligned}
\tag{2}
$$

where the ω's are parameters to be estimated. The corresponding input share equations are derived by obtaining the partial differentiation of (2) with respect to lnP_L and lnP_K i.e.

$$
S_i = \frac{\partial lnC}{\partial lnP_i} = \omega_i + \sum_{j=1}^{2} \omega_{ij}\ lnP_j + \omega_{iQ}\ Q + \omega_{iT}\ T \tag{3}
$$

In many applications of flexible functional forms some of the regularity conditions prescribed by economic theory are not supported by the empirical results.[3] Burrell (1989), in a review of models of agricultural factor demand, states that many of the restrictions implied by duality theory are rarely found to hold globally and en bloc in empirical models of agricultural production. Examples of such cases include Lopez (1982), Shumway (1983), McKay, Lawrence and Vlastuin (1983), Higgins (1986) and Wall and Fisher (1987). When this happens "it casts doubt not only on the assumption of constrained optimisation at the micro level but also on a number of other features of the maintained hypothesis" (Applebaum (1979)). As a result, the following restrictions associated with the regularity conditions of the cost function are imposed during estimation

(1) Linear homogeneity in input prices (i.e. the shares add to 1): $\sum_{i=1}^{2} \omega_i = 1$, $\sum_{j=1}^{2} \omega_{ij} = 0$, & $\sum_{i=1}^{2} \omega_{iQ} = 0$.

[3] The regularity conditions for a cost function are outlined by Mandy (2000) amongst others.

(2) Symmetry: $\omega_{ij} = \omega_{ji}$.

(3) Linear homogeneity in output: $\omega_Q = 1$, $\omega_{QQ} = 0$ & $\omega_{QT} = 0$.

(4) Homotheticity: $\omega_{iQ} = 0 \; \forall$ i.

While these restrictions are imposed during the estimation stage, the validity of the imposition may of course be tested through standard likelihood ratio (LR) tests. One further restriction which is tested for, is a test of *labour augmenting technical progress* as presented by Hall and Nixon (1999). Labour augmenting technical progress is equivalent to a corresponding increase in the labour force. Under this form of technical progress, it implies that a given output can be obtained from a given capital input combined with a labour input that decreases as time progresses. One unit of labour does as much as say two units used to do. The test for labour augmenting technical progress necessitates that the coefficients on the trend term T be replaced by the product of different coefficients on the price of labour P_L and an additional parameter - κ.

(5) $\omega_T = \kappa \ * \omega_L$, $\omega_{iT} = \kappa \ * \omega_{Li}$, $\omega_{QT} = \kappa \ * \omega_{LQ}$ & $\omega_{TT} = \kappa \ * \omega_L$

Two commonly reported sets of results associated with systems estimation such as (2) and (3) are the Allen-Uzawa partial elasticity of substitution (σ_{ij}) and the price elasticity of demand (η_{ij}). For the translog, the relevant expressions for both sets of long-run (superscript l) elasticities are

$$\sigma^l_{ii} = \frac{\omega_{ii} + S^2_i - S_i}{S^2_i}, \quad \sigma^l_{ij} = \frac{\omega_{ii} + S_i S_j}{S_i S_j} \tag{4}$$

and

$$\eta^l_{ii} = S_i \sigma^l_{ii} \quad \eta^l_{ij} = S_j \sigma^l_{ij} \tag{5}$$

Note that while some regularity conditions associated with the cost function are imposed, concavity in input prices is merely examined ex-post. All estimation results along with the LR test of labour augmenting technical progress and the elasticity estimates are presented in section 4.

3 A Dynamic Model of Irish Factor Demands

The previous section outlined a static long-run model of an econoomy's supply-side. In this section, an introduction to the Allen & Urga (1999), Allen & Urga (1995) dynamic cost function is provided along with an application in an Irish context. Anderson and Blundell (1982) had already specified the following ARDL(1,1) dynamic share equation

$$S_t = M_1 S_t^E + M_2 S_{t-1}^E + M_3 S_{t-1} \tag{6}$$

where $M_1 + M_2 + M_3 = I$ the identity matrix. Expressing (6) in error correction form yields the following

$$\triangle S_t = N \triangle S_t^E + R\left(S_{t-1}^E - S_{t-1}\right) \tag{7}$$

where $N = M_1, R = I - M_3 = M_1 + M_2$. Therefore, the disequilibria in n-1 factor shares at time t-1 affects the contemporaneous period adjustment. However, the sum of the changes in shares must be equivalent to zero

$$\sum_i^n \left(S_i - S_i^E\right)_{t-1} = 0 \tag{8}$$

which implies that the columns of the R matrix must total zero. Given that there are only $n-1$ independent dis-equilibria shares, the R matrix is of reduced rank: $(n-1)*(n-1)$. This results in an identification problem whereby only the *ratio* of individual adjustment coefficients (r_{ii}/r_{ii}) are retrievable not the individual coefficients themselves. Consequently, indicators such as the return to scale or the rate of neutral technical progress cannot be calculated using the Anderson and Blundell (1982) specification alone. Furthermore, it is not possible to measure any of the short-run elasticities.[4] The contribution of Allen and Urga (1995) and (1999) was to suggest that the joint estimation of the factor shares along with the underlying

[4] For a full discussion on this point see Allen and Urga (1999) and Urga and Walters (2003).

dynamic cost function would overcome this problem. The additional constraint used to identify the components of the R matrix is that the sum of the input price times input quantity equals total cost levels each time period. Allen and Urga (1995) and (1999) propose the following cost function where the N matrix in (7) is mapped onto the scalar δ[5]

$$\begin{aligned} lnC_t = \delta lnC_t^E + (1-\delta)\sum_{i=1}^{2} S_{it-1} lnP_{it} + (1-\delta)(\sum_{i=1}^{2} S_{i,t-1} lnP_{it} \\ -\sum_{i=1}^{2} S_{i,t-1}^E lnP_{it-1}) + \sum_{i=1}^{2}\sum_{j=1}^{2} m_{ij}\left(S_{j,t-1}^E - S_{j,t-1}\right) lnP_{it} \end{aligned} \tag{9}$$

where the superscript E denotes the equilibrium value of either total cost or the factor input share and the parameters m_{ij} are the elements of the M_2 matrix in (6). As noted by Allen and Urga (1999), in equilibium, $S_{it} = S_{it}^E$. This arises in two different cases, one where there is constant factor prices - $\triangle lnP_{it} = 0$ or where there is a constant increase in price - $\triangle lnP_{it} = \lambda$. In the former case $lnC_t = lnC_t^E = lnC_{t-1}^E$ while in the latter, $lnC_t^E = lnC_{t-1}^E + \lambda$. Using the envelope theorem, the following input share is obtained

$$\frac{\partial lnC_t}{\partial lnP_t} = S_t = \delta S_t^E + (1-\delta)S_{t-1} + M_2\left(S_{t-1}^E - S_{t-1}\right) \tag{10}$$

This can then be re-parameterised in accordance with the Anderson and Blundell (1982) error correction model (7). Re-specifying (10)/(7) on a single-equation basis yields the following

$$\triangle S_{it} = \delta \triangle S_{it}^E + \sum_{j=1}^{2} r_{ij}\left(S_{jt-1}^E - S_{jt-1}\right) \tag{11}$$

with r_{ij} being the elements of the R matrix and $R = \delta I + M_2$. Jointly estimating (11) and (9) overcomes the identification issue signaled by Anderson and Blundell

[5]Urga and Walters (2003) provide another application of the model.

(1982). Furthermore, the addition of the cost function to the suite of estimated equations improves the efficiency of the final parameters. Nested within (11) are different permutations of an error correction mechanism. For instance if M_2 is a diagonal matrix then (11) becomes a *simple independent adjustment error correction mechanism* (Allen and Urga (1999) and Urga and Walters (2003)). This would also correspond to R being diagonal in (7). Given the error correction specification for (7) and (11), short-run price elasticities may then be derived for the factor demands. These are differentiated from the short-run equivalents in (5) by the superscript s.

$$\eta_{ii}^{s} = \frac{\delta\omega_{ii}}{S_i} + S_i - 1 \quad \eta_{ij}^{s} = \frac{\delta\omega_{ij}}{S_i} + S_j \tag{12}$$

Given the presence of both long-run and short-run elasticities, one issue which may also be examined is whether or not the Le Chatelier principle is observed in this case. The Le Chatelier principle states that short-run elasticities must be smaller in absolute terms then their long-run equivalents. It can be established that in order for

$$\eta_{ii}^{l} - \eta_{ii}^{s} > 0 \tag{13}$$

to be the case, then the following must hold

$$\omega_{ii}\left(\delta - 1\right) > 0 \tag{14}$$

if the elasticities are given by (12). If the principle holds, it provides considerable justification for the dynamic approach adopted here. The following section presents the results of the estimations, elasticity estimates and LR tests.

4 Data and Empirical Results

All data used in the analysis is taken from the interpolated series of national income accounts prepared at the Central Bank of Ireland. Estimation was conducted over the period 1981:1 to 1999:4. Output is in constant 1995 prices and labour is measured as the actual numbers of people employed. Wages are derived by dividing total compensation of employees in the economy by labour. The derivation of capital and the cost of capital is presented in Appendix A to this paper. Full details of the data along with details of its interpolation may be obtained from McGuire, O'Donnell and Ryan (2002). All estimations were conducted using the nonlinear systems estimator (NLSYSTEM) in WinRats-32 5.0.[6]

Table 1 presents two sets of LR tests. The first is for labour augmenting technical progress. Note, this test was only conducted on the static model ((2) & (3)). From the Table it is apparent, that the null hypothesis that technical progress in the Irish economy between 1981 and 1999 was labour augmenting can be rejected. In it itself, this is quite an interesting result, as it is not in keeping with most supply-side applications which tend to impose Harrod-neutral technical progress as a stylised fact. For instance, if it emerged that technical progress in Ireland has been capital and not labour saving, then technical progress would not be a cause of systemic structural unemployment in an Irish context.[7] The second LR test examines whether the static model is a valid restriction of the dynamic model.[8] The calculated χ^2 would suggest that it is not, thereby validating the decision to proceed with the dynamic structure.

Table 2 presents the parameter results for both the long-run static model of (2) & (3) and the short-run dynamic model presented in (9) & (11). Parameter values for the long-run cost and share functions are obtained from the dynamic system by substituting the expressions in (3) & (4) into the relevant equilibrium expressions in (9) & (11). Overall, both the static and dynamic systems have a relatively large number of parameters significant at the 1 per cent level (73% and 75% respectively). In terms of the speed of adjustment of input usage, the parameter of interest is δ. Urga and Walters (2003) have defined this parameter as *the inverse of the speed of dynamic adjustment*. A simple t-test reveals that the estimate of δ in this case (1.045) is statistically significant from 0.[9] Thus, given the parameter estimates in Table 2 and the value of δ, the LeChatelier principle holds in this case, suggesting that there is indeed a rigidity in the response of factor inputs to changes in prices. However, obtaining the inverse of δ, suggests that up to 96% of the long-run changes in the demand for labour and capital in the Irish economy due to a price change occur

[6] All programs are available from the author upon request.

[7] Boskin and Lau (2000) present a generalised framework for the testing of the structure of economic growth which in turn reveals the magnitudes and biases of technical progress.

[8] This is effectively a test of whether the following can be imposed on the dynamic model: $\delta = 1$, $m_{11} = m_{12} = m_{21} = m_{22} = 0$.

[9] The standard error of the δ parameter is 0.753e-3.

in the year of the actual price change. This compares with a speed of adjustment of 90% in Urga and Walters (2003) and 75% in Allen and Urga (1999). The result adds some weight to the criticism of Hall and Nixon (1999) of the Allen and Urga (1999) dynamic approach. They claim that the dynamic response from these systems can be "implausibly fast".

The long and short run elasticities are summarised in Table 3. It is evident that labour and capital are substitutes, as one would have expected. As verified by (14), all long-run price elasticities are greater in absolute terms then their short-run equivalents. Concavity in input prices is observed in all cases except for the long-run price elasticity of labour. However, the Allen-Uzawa partial elasticity of labour σ_{LL} is positive. In terms of the magnitudes of the results, it should be noted that the long-run elasticities achieved are quite large relative to other results. For instance, the Allen-Uzawa partial elasticity for capital is over three times as large as that achieved by Hall and Nixon (1999). Similarly, the price elasticities in Table 3 are over four times as large as those in Hall and Nixon (1999).[10]

In terms of technical progress, estimates of both the nature and scale of the rate of technical progress can be inferred from the dual cost function. In the case of the static cost function, both linear homogeneity in output and homotheticity are imposed during estimation. Consequently, certain equivalent relationships hold between dual and primal measurements of technical progress. Namely, the rate of cost diminution $\phi\left(P_l, P_K, Q, T\right)$[11] is equivalent to the rate of technical progress and evidence of cost neutrality is equivalent to evidence of Hicks-neutral technical progress.[12] From Chambers, cost neutrality holds if

$$\frac{\partial lnS_1(P_1,Q,T)}{\partial T}=\frac{\partial lnS_2(P_2,Q,T)}{\partial T} \tag{15}$$

From Table 2, it is evident that this holds in both static and dynamic cases. Thus, any technical progress in an Irish context during the 1980's and 1990's period is

[10] It should be noted however, that the model used in Hall and Nixon (1999) is not exactly comparable to that in this paper. They present a dynamic equation in factor *levels* rather than in *shares*.

[11] Given by $\partial lnC/\partial T$

[12] See Chambers (1988) for more on these primal-dual relationships.

synonomous with vertical, upward shifts of the production function leaving the marginal rate of substitution between capital and labour independent of time. Recall, that the specific test for labour augmenting technical progress was rejected by the data. Hicks-neutral technical progress can be regarded as being *equal-factor augmenting.*

For the dual case, the relationship between the rate of cost diminuition and the rate of technical progress $TP\left(L,K,T\right)$ is given by the following

$$TP\left(L,K,T\right)=-\epsilon\left(P_l,P_K,Q,T\right)\phi\left(P_l,P_K,Q,T\right) \tag{16}$$

where $\epsilon\left(P_l,P_K,Q,T\right)$ is the elasticity of size.[13] Under the imposition of linear homogeneity in output, $\epsilon=1$ and hence $TE\left(\right)$ and $\phi\left(\right)$ are equivalent. This applies in the case of the static cost function. In the dynamic case, the elasticity of size is now equal to $\delta\omega_Q$ which has been established as not being equal to 1. Thus, in order to arrive at the equivalent expression for technical progress, the rate of cost diminuition must be scaled by δ. The different annual rates are plotted in Figure 1, with the linear rates for the static cost function contrasting with the non-linear estimates of the dynamic approach. Over the total annual sample (1981-1999), the average rate of progress differs by over 1.2 per cent 1.07 per cent (Static) versus 2.29 per cent (Dynamic). For the latter part of the sample however (1993-1999), the rates of progress are quite similar (2.66 per cent (Static) versus 2.80 per cent (Dynamic)). In modelling the supply side of the UK economy, Hall and Nixon (1999) report an annual estimate of 2.3 per cent technical growth.

Diagnostic tests for both the static and dynamic systems are presented in Table 4. While heteroscedastic errors are not a problem (except for the static cost function), there would appear to be significant autocorrelation in the error structure of both the static and dynamic frameworks. Both the static and dynamic systems were re-estimated with the contemporaneous and lagged values of the right-hand side variables[14] added to each function. However, there was no improvement in the serial correlation observed in the estimated residuals. Serial correlation is not an uncommon feature of systems such as the one estimated here. Urga and Walter-

[13]Given by $\partial lnC/\partial lnQ$

[14]Logs of output, wages and the interest rate

s (2003) for example, report similiar diagnostics for a static system with both a translog and a logit functional form. Cipollini, Hall and Nixon (2000) in an application of a similar dynamic system also report serial correlation. They suggest that the presence of serial correlation in such a system may be indicative of time varying technical progress or possibly endogenous scrapping. Given the rapid growth rates experienced in the Irish economy between 1995 and 1999, it is conceivable that technology has progressed in a non-linear manner over the time-period.

5 Conclusions

This paper has applied the Allen & Urga (1995) and Allen & Urga (1999) model of dynamic factor demands to quarterly Irish national income data between 1981 and 1999. The model is sufficiently flexible to allow for the nesting of a long-run static model of factor demands within a dynamic framework. The use of a dynamic framework is a more plausible specification given the obvious rigidities confronting any potential alterations in the application of factor inputs such as capital and labour. Furthermore, the derivation and estimation of both static and dynamic systems enables the calculation of short and long-run elasticities as well as the estimation of a parameter denoting the speed of dynamic adjustment in the use of factor inputs.

The results indicate that the use of a dynamic specification is indeed warranted in an Irish case. A specific test for labour augmenting technical progress in Irish supply-side models is not supported by the data. Rather, evidence of Hicks-neutral equal factor augmenting technical progress is found. Technical progress over the 1993-1999 time period averages at 2.7 per cent per annum between the static and dynamic approaches. The Le Chatelier Principle is observed in an Irish context, and concavity in input prices holds in all cases except for the long-run case of labour. However, the magnitude of the long run elasticities are quite large when compared with similar empirical applications. The speed of adjustment in Irish input usage is very rapid, which is not an uncommon feature of the Allen & Urga (1995) approach. In fact the almost universally estimated rapid response of input usage in applications of the model, has led to an alternative specification by Hall and Nixon (1999). Rather then specify adjustments in input shares, Hall and Nixon

(1999) examine changes in the actual levels of factor inputs. Future studies may wish to apply this alternative approach.

Diagnostic tests of the two models reveals the presence of serial correlation in the error structure. This is not an uncommon feature of economy-wide supply-side systems. In particular, the rapid growth rate of the Irish economy throughout the 1990s may have resulted in significant changes in the rate of technical progress experienced. Future work may seek to address this issue by allowing for non-linear changes in technical progress in the present dynamic system.

References

Allen, C. and Mestre, R.: 1997, Estimating the supply-side of the EMI multi-country model: Data and specification issues, *EMI Internal Paper MESD S316297*, European Monetary Institute.

Allen, C. and Urga, G.: 1995, Derivation and estimation of interrelated factor demands from a dynamic cost function, *Discussion Paper 10–95*, London Business School, Centre for Economic Forecasting.

Allen, C. and Urga, G.: 1999, Interrelated factor demands from dynamic cost functions: An application to the non-energy business sector of the UK economy, *Economica* **66**, 403–13.

Anderson, G. and Blundell, R.: 1982, Estimation and hypothesis testing in a dynamic singular equation systems, *Econometrica* **50**, 1559–1571.

Applebaum, E.: 1979, On the choice of functional forms, *International Economic Review* **20**, 449–58.

Boskin, M. and Lau, L.: 2000, Generalized Solow-Neutral technical progress and post-war economic growth, *Working Paper w8023*, National Bureau of Economic Research, Cambridge, MA.

Breusch, T.: 1978, Testing for autocorrelation in dynamic linear models, *Australian Economics Papers* **17**, 334–355.

Burrell, A.: 1989, The demand for fertiliser in the United Kingdom, *Journal of Agricultural Economics* **40**, 1–20.

Chambers, R.: 1988, *Applied Production Analysis The Dual Approach*, Cambridge University Press, Cambridge, UK.

Cipollini, A., Hall, S. and Nixon, J.: 2000, The inter-relationship bewteen monetary policy and endogenous technical progress, *Discussion Paper 2000.05*, Centre for International Macroeconomics, University of Oxford.

Engle, R.: 1982, Autoregessive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* **50**, 987–1008.

Godfrey, L.: 1978, Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables, *Econometrica* **46**, 1293–1302.

Gundlach, E.: 2001, Interpreting productivity growth in the new economy: Some agnostic notes, *Working paper*, Kiel Institute of World Economics, Dusternbrooker Weg 120 24105 Kiel.

Hall, S. and Nixon, J.: 1999, A consistent approach to modelling dynamic factor demands, *Discussion paper*, The Centre for Economic Forecasting, London Business School, London, NW1 4SA.

Higgins, J.: 1986, Input demand and output supply on Irish farms - A microeconomic approach, *European Review of Agricultural Economics* **13**, 447–93.

Jorgenson, D. W., Gollop, F. and Fraumeni, B.: 1987, *Productivity and U.S. Economic Growth*, Harvard University Press, Cambridge MA.

Lopez, R.: 1982, Applications of duality theory to agriculture, *Western Journal of Agricultural Economics* **7**, 353–66.

Mandy, D.: 2000, Economics 405: Advanced MicroEconomic Theory I Course Packet, University of Missouri-Columbia.

McGuire, M., O'Donnell, N. and Ryan, M.: 2002, Interpolation of quarterly data for ECB/NCB multicountry modelling exercise - Data update to 1999q4, *Research paper*, Central Bank of Ireland, Dame Street, Dublin 2.

McKay, L., Lawrence, D. and Vlastuin, C.: 1983, Proft, output supply and input demand functions for multiproduct firms: The case of Australian agriculture, *International Economic Review* **24**, 323–39.

Shumway, C.: 1983, Supply, demand and technology in a multiproduct industry: Texas fieldcrops, *American Journal of Agricultural Economics* **65**, 748–60.

Slevin, G.: 2001, Potential output and the output gap in Ireland, *Technical Paper RT/5/2001*, Central Bank of Ireland, Dame Street, Dublin 2.

Urga, G. and Walters, C.: 2003, Dynamic translog and linear logit models: A factor demand analysis of interfuel substitution in US industrial energy demand, *Energy Economics* **25**, 1–21.

Wall, C. and Fisher, B.: 1987, Modelling a multiple output production system: Supply response in the Australian sheep industry, *Technical Report Research Report 11*, Department of Agricultural Economics, University of Sydney, University of Sydney.

A Capital and the Cost of Capital

Almost all data used in this paper is taken from the interpolated series of national income accounts maintained at the Central Bank. However, certain adjustments have been made to the capital and cost of capital series.

The Irish housing market experienced a sharp increase in prices throughout the latter part of the 1990's and the early part of the new century. Many reasons have been advanced for this including institutional factors which resulted in a relatively slow adjustment in the Irish housing stock to these new market conditions. Consequently, non-housing capital is used as the total capital stock. A value of housing stock was obtained for the initial time-period of 1980:1 and an associated non-housing capital stock generated. This was rolled forward using the perpetual inventory method with a non-housing investment series.

The cost of capital series (CC0) is based on the standard Jorgenson, Gollop and Fraumeni (1987) expression

$$\mathrm{CC0}_t = \mathrm{I}_t[\mathrm{r}_t + \sigma_t - (\mathrm{I}^e_{t+1} - \mathrm{I}_t)/\mathrm{I}_T] \tag{17}$$

where I is an investment deflator, e denotes expected value, r_t is the cost of borrowing funds and σ_t is a depreciation factor. Two adjustments were made to this series. Firstly, a split depreciation schedule was used with the level of depreciation increasing from 6.25 per cent prior to 1996 to 9 per cent thereafter. This, in part, reflected the changing nature of the Irish capital stock with anecdotal and investment evidence of movements towards a faster depreciating stock. It also reflected the exclusion of housing from the capital stock. The depreciation rate in previous applications had been 4 per cent. This rate appeared quite low, particularly, when compared with rates used by the Bureau of Economic Analysis (BEA) in the United States.

The second adjustment made to (22) was to increase the cost of borrowing funds. Previously, r_t had been equivalent to the AAA or prime rate - the rate charged to large commercial customers for short-term borrowings. However, a simple average of the AAA rate and the AA rate is now used. The latter is the rate to charged to more medium sized enterprises. Given, that these enterprises usually face larger rates, the new r_t is persistently above the level of the older rate.

Table 1: Likelihood Ratio Tests

	1	2	3		
	Static	Static	Dynamic	1 v 2	2 v 3
Log Determinant	-23.226	-27.210	-45.43		
Lab. Augmenting Technical					
Progress Imposed	$\surd$	X	X		
χ^2				235.010	1149.52
P-Value				0.00	0.00

Sample size = 76

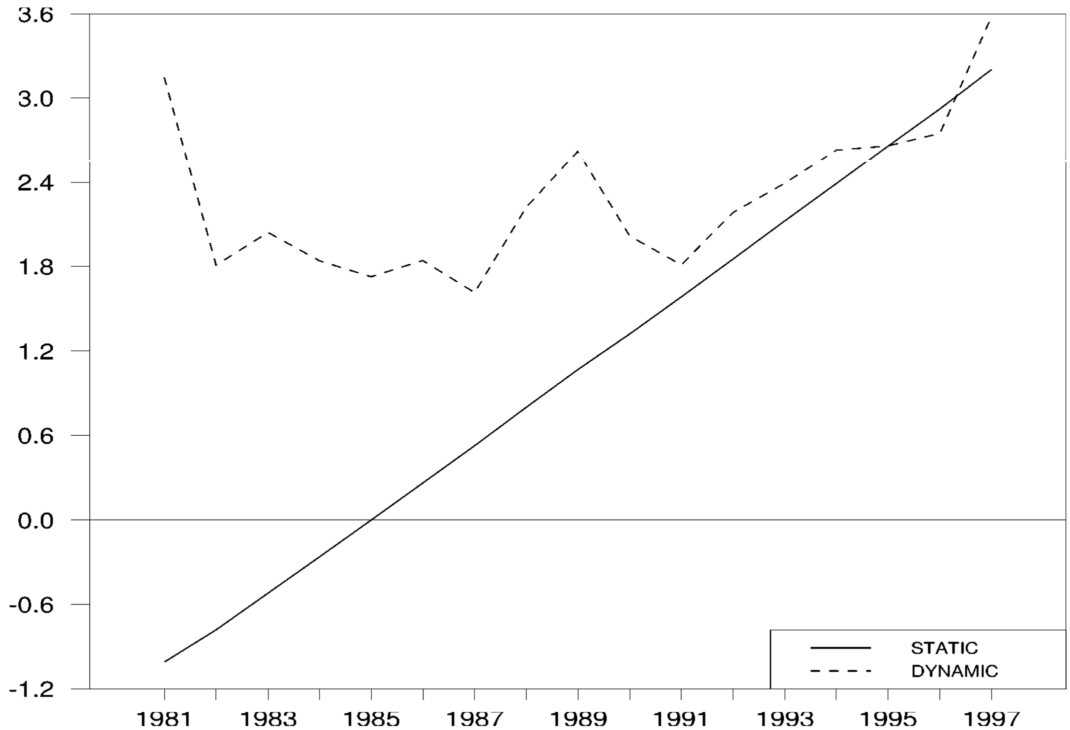

Figure 1: Technical Progress % Growth

Table 2: Static and Dynamic Parameter Estimates

	Static Model		Dynamic Model	
Parameter	Estimate	P-Value	Estimate	P-Value
ω_0	9.722	0.000	9.93	0.000
ω_Q	1.000	0.000	1.000	0.000
ω_T	-0.001	0.000	-0.018	0.000
ω_1	-0.476	0.000	-6.801	0.000
ω_2	1.476	0.000	7.801	0.000
ω_{11}	0.248	0.000	0.141	0.000
ω_{12}	-0.239	0.000	-0.141	0.000
ω_{22}	0.230	0.000	0.141	0.000
ω_{1Q}	0.000	0.302	-3.522e-07	0.995
ω_{2Q}	0.000	0.999	-3.419e-07	0.996
ω_{1T}	-0.133e-03	0.169	0.086	0.143
ω_{2T}	-0.886e-03	0.000	-0.086	0.143
ω_{QT}	0.000	0.004	0.000	0.000
ω_{QQ}	0.000	0.999	0.000	0.000
ω_{TT}	0.198e-03	0.000	-2.49e-05	0.449
δ			1.045	0.000
m_{11}			15.776	0.000
m_{12}			16.807	0.000
m_{21}			-16.834	0.000
m_{22}			-17.864	0.000

Sample size = 76

Table 3: Elasticity Results

Elasticity	Long-Run	Short-Run
σ_{LL}	0.746	
σ_{LC}	2.094	
σ_{CC}	-2.307	
η_{LL}	0.479	-0.045
η_{LC}	0.739	0.040
η_{CC}	-0.814	-0.079
η_{CL}	1.295	0.074

Note: All elasticities are evaluated at the sample mean.

Table 4: Miss-Specification Tests - P-Values

	AR(1)	AR(4)	ARCH
Static :			
Total Cost	0.003	0.000	0.000
Labour	0.000	0.000	0.947
Capital	0.000	0.000	0.885
Dynamic :			
Total Cost	0.000	0.000	0.104
Labour	0.022	0.024	0.177
Capital	0.022	0.024	0.177

Note: AR tests are from Godfrey (1978) and Breusch (1978), while the ARCH tests are from Engle (1982).